SPARSE BAYESIAN METHODS FOR CONTINUOUS SPEECH RECOGNITION

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The prominent modeling technique for speech recognition today is the hidden Markov model with Gaussian emission densities. However, they suffer from an inability to learn discriminative information. Artificial neural networks have been proposed as a replacement for the Gaussian emission probabilities under the belief that the ANN models provide better discrimination capabilities. However, the use of ANNs often results in over-parameterized models which are prone to overfitting. Techniques such as cross-validation have been suggested as remedies to the overfitting problem but employing these is wasteful of both resources and computation. Further, cross-validation does not address the issue of model structure and over-parameterization.

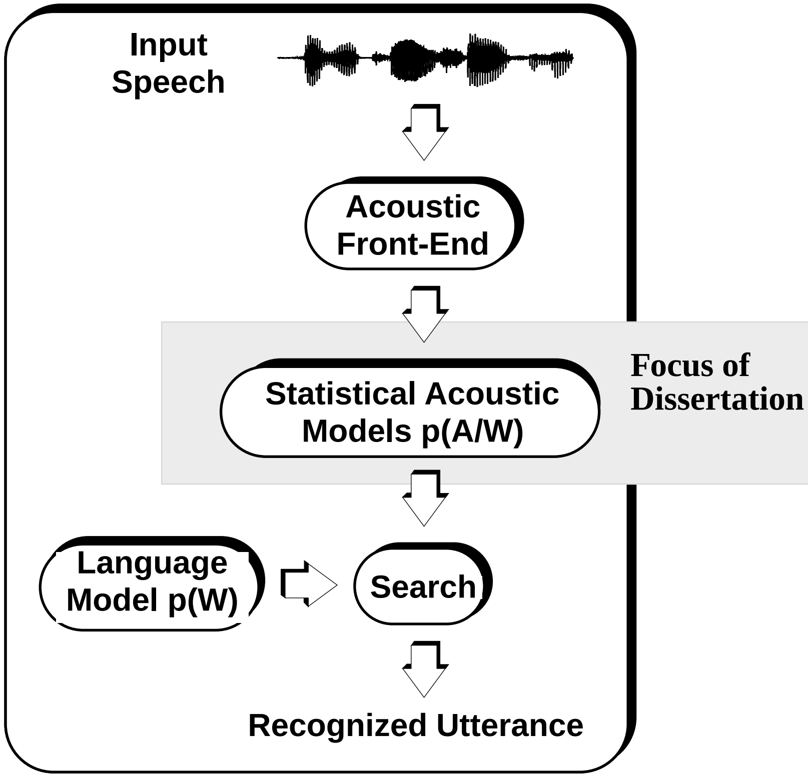
Recent work on machine learning has moved toward automatic methods for controlling generalization and parameterization. A model that has gained much popularity recently is the support vector machine (SVM). SVMs use the principle of structural risk minimization to simultaneously control generalization and performance on the training set. A recent dissertation from this university has employed the SVM in a hybrid framework for speech recognition. While the HMM/SVM hybrid produced a decrease in the error rate, the implementation had some significant shortfalls which we hope to address in this work. First, the SVMs are not probabilistic in nature and, thus, are not able to adequately express the posterior uncertainty in predictions. This is particularly important in speech where there is significant overlap in the feature space. The SVMs also make unnecessarily liberal use of parameters to define the decision region.

In this dissertation, we study a Bayesian model which takes the same form as the SVM model. This model, termed the relevance vector machine (RVM), provides a fully probabilistic alternative to the SVMs. The RVMs have been found to provide generalization performance on par with SVMs while typically using nearly an order of magnitude fewer parameters. Sparseness of the model is automatic using MacKay's automatic relevance determination methods. In this work we propose to develop the first speech recognition system using RVMs. Similar to hybrid HMM/ANN systems, the RVM model will replace the Gaussian density in the HMM models. To accomplish this, we must develop closed-loop training routines which insure convergence and optimality. Computational issues make this an impossibility currently and must be addressed before a scalable system is feasible.

ORGANIZATION OF PRESENTATION

- **Problem Definition**: Speech recognition and the acoustic modeling problem
- **Prior Art**: Hidden Markov models and artificial neural network hybrid systems
- **Prior Art**: Support vector machines, hybrid systems
- **Proposed Methodology**: Bayesian methods, automatic relevance determination and the relevance vector machine
- **Proposed Work:** Preliminary experiments and proposed experiments

ASR PROBLEM



- The Front-end maintains information important for modeling in a reduced parameter set.

- The language model typically predicts a small set of next words based on knowledge of a finite number of previous words (N-grams) — leads to search space reduction.

Bayesian formulation for speech recognition:

$$P(W|A) = \frac{P(A|W)P(W)}{P(A)}$$

Objective: minimize the word error rate by maximizing $P(W|A)$

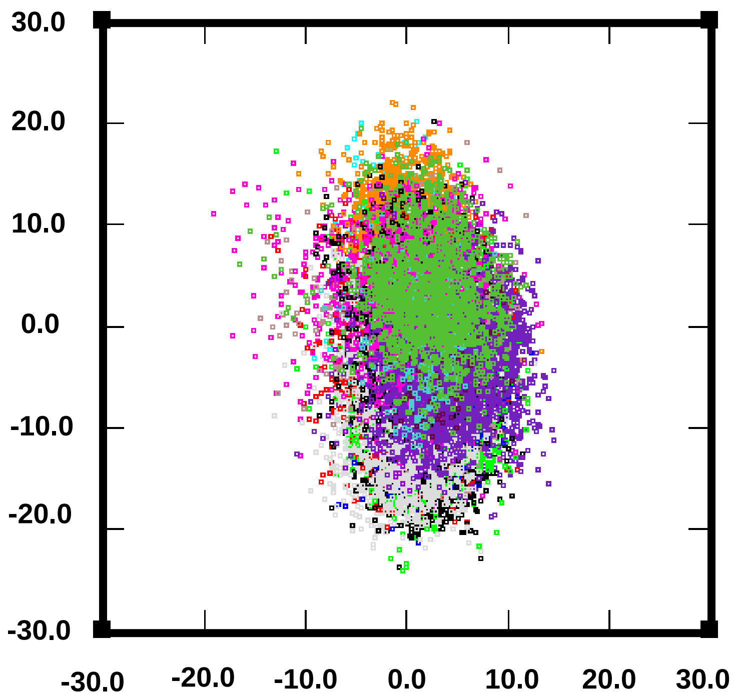
Approach: maximize $P(A|W)$ (training)

Components:

- $P(A|W)$: acoustic model (hidden Markov models, mixture of Gaussians)
- $P(W)$: language model (statistical, N-grams, finite state networks)
- $P(A)$: acoustics (ignore during maximization)

ACOUSTIC MODELING

Acoustic Confusability: Requires reasoning under uncertainty!



- First two cepstral coefficients for all vowels (based on a conversational speech corpus — SWITCHBOARD).

- Overlap represents a fundamental barrier for good classification.

Acoustic Models Must:

- Model the temporal progression of the speech signal

- Model the acoustic characteristics of sub-word units

- Account for variations in speaker characteristics (speaker-independent)

Acoustic Models Should:

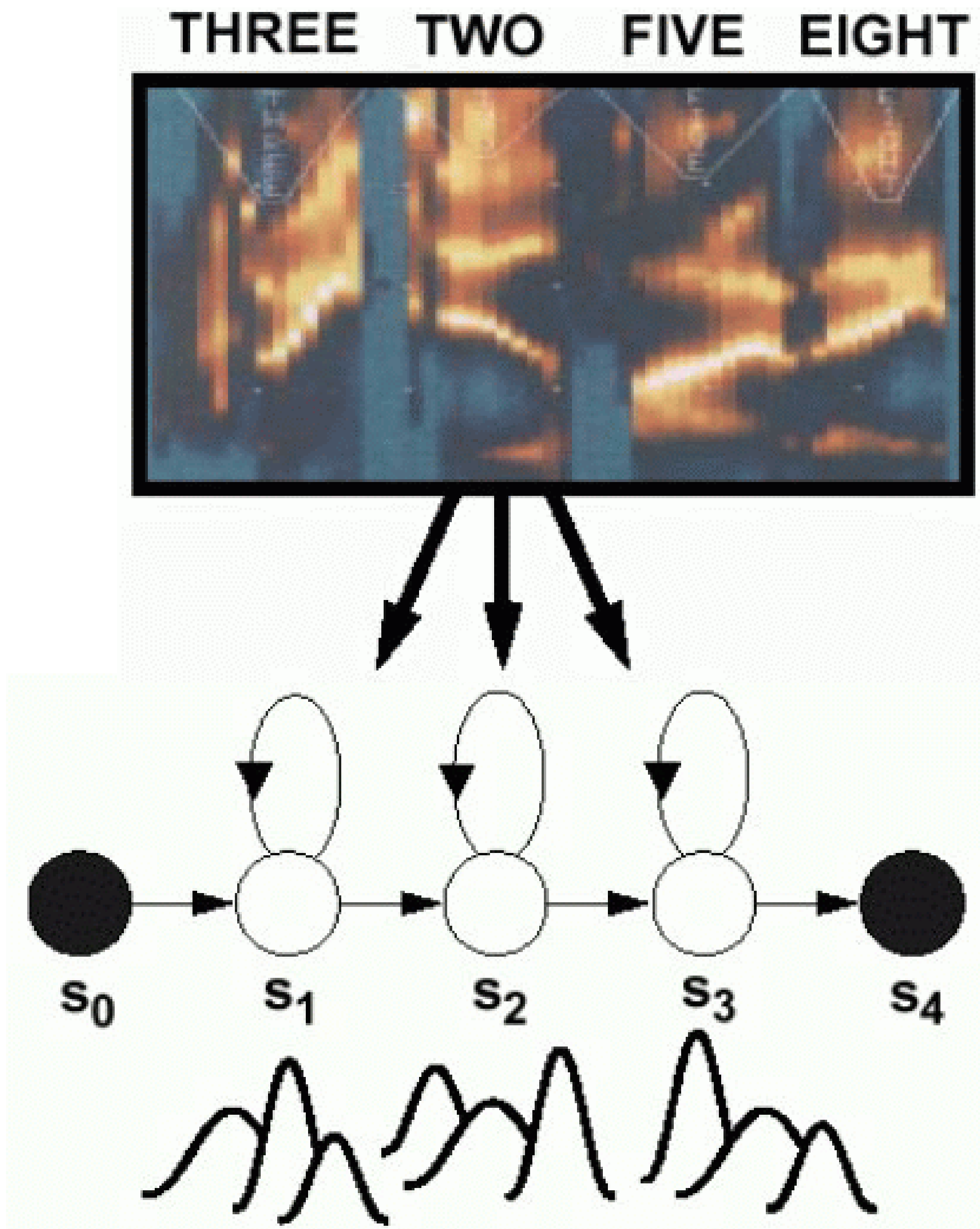
- Optimally trade-off discrimination and representation

- Make efficient use of parameters

- Produce confidence measures of their predictions for higher-level decision processes

PRIOR ART: HMMs

- Acoustic models encode the temporal evolution of the features (spectrum).
- Gaussian mixture distributions are used to account for variations in speaker, accent, and pronunciation.
- Sharing model parameters is a common strategy to reduce complexity.
- The goal of our research is to replace the Gaussian likelihood computation at each state with a machine that incorporates notions of:
  - **discrimination ("one vs. all")**
  - **Bayesian statistics (priors)**
  - **confidence**
  - **sparsity**
- Maintain computational efficiency?



PRIOR ART: HMMs

- Data-driven modeling supervised only from a word-level transcription.
- The expectation/maximization (EM) algorithm is used to improve our estimates:

  $$\log P(Data|\bar{\lambda}) \geq \log P(Data|\lambda)$$

  if:

  $$Q(\lambda, \bar{\lambda}) \geq Q(\lambda, \lambda)$$

  Approach: maximum likelihood estimation
- Computationally efficient training algorithms (Forward-Backward) have been crucial.
- Batch mode parameter updates are typically preferred.
- Decision trees are used to optimize sharing parameters, minimize system complexity, and integrate additional linguistic knowledge.

- Initialization
- Single Gaussian Estimation
- 2-Way Split
- Mixture Distribution Reestimation
- 4-Way Split
- Reestimation

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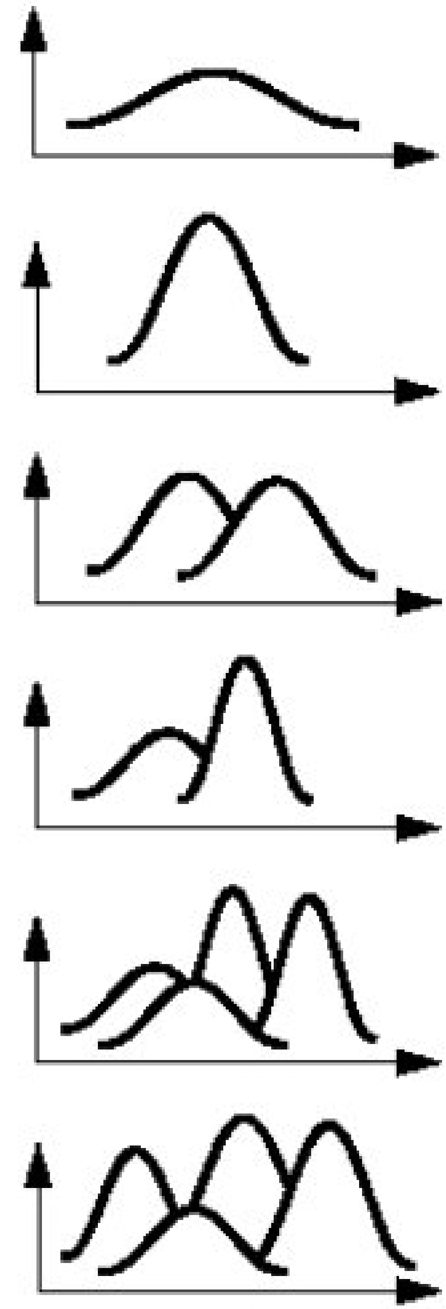
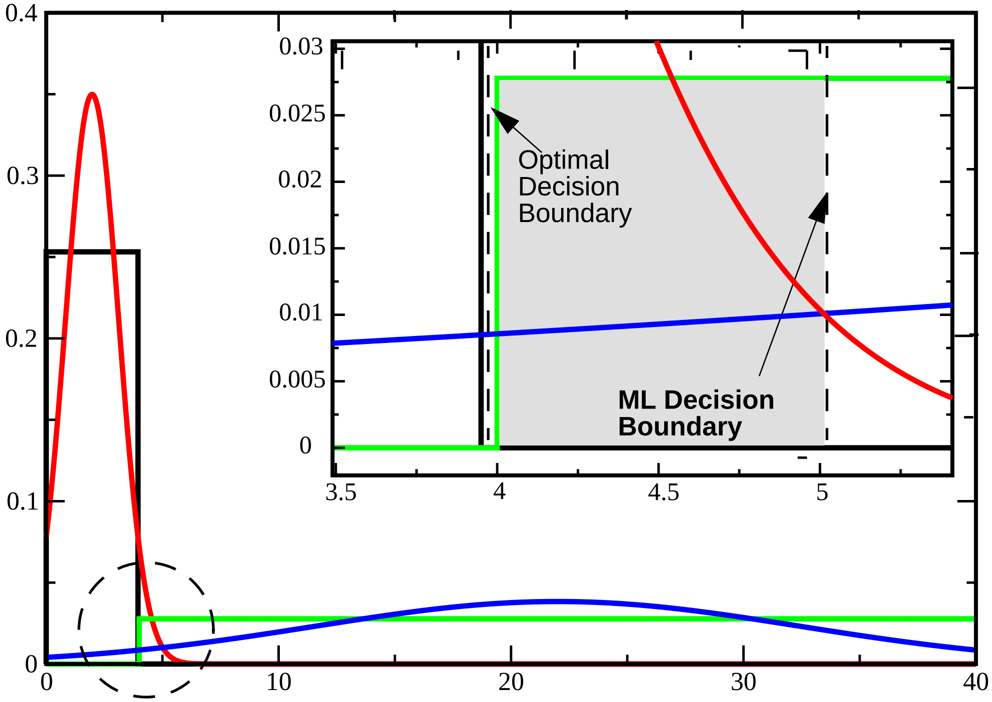
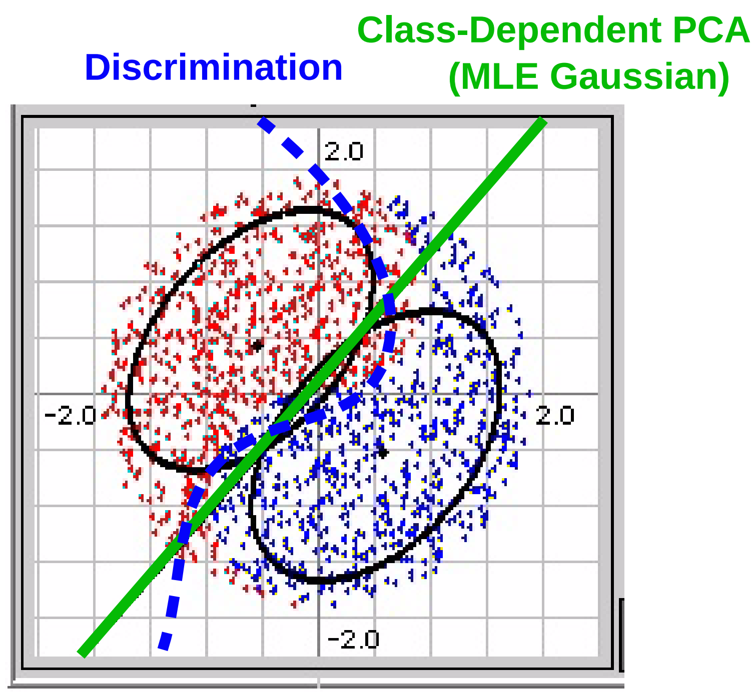


PRIOR ART: HMMs

Convergence in maximum likelihood does not translate to optimal classification:



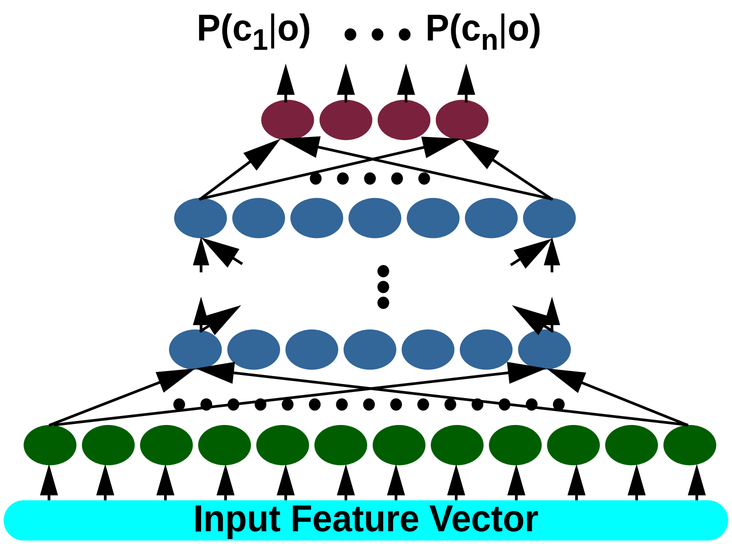
- Error results from fitting uniform distributions with Gaussians (and using an ML boundary).
- Since the classes are separable, finding the optimal decision surface is trivial.



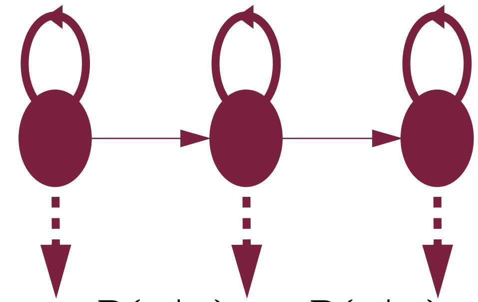
- Data not separable by a hyperplane (a nonlinear classifier is needed).
- Gaussian MLE models tend towards the center of mass (overtraining).

Solution: Nonlinear discriminative classifiers!
First Cut: Artificial Neural Networks

PRIOR ART: HMM/ANN HYBRIDS



ANN Replaces Mixture Gaussian Emission Probability



$$\frac{P(\mathbf{o}|c)}{P(\mathbf{o})} = \frac{P(c|\mathbf{o})}{P(c)}$$

Architecture:

- ANN provides flexible, discriminative classifiers for emission probabilities that avoid the HMM independence assumptions (can use wider acoustic context).
- Trained using Viterbi iterative training (hard decision rule) or can be trained to learn Baum-Welch targets (soft decision rule).

Shortcomings:

- Prone to overfitting: require cross-validation to determine when to stop training. **Need a method for automatically penalizing overfitting!**
- No substantial recognition improvements over HMM/GMMs

RISK MINIMIZATION

- Expected Risk:

$$R(\alpha) = \int \frac{1}{2}|y - f(x, \alpha)|\, dP(x, y)$$

Not possible to estimate $P(x, y)$.

- Empirical Risk Minimization:

$$R_{emp} = \frac{1}{2l}\sum_{i=1}^{l}|y_i - f(x_i, \alpha)|$$

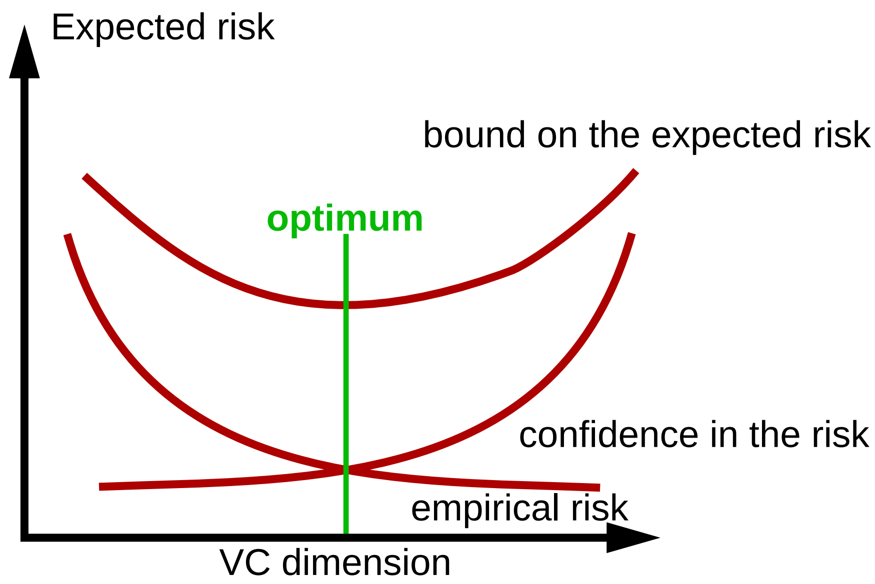
- Related by VC (Vapnik-Chervonenkis) dimension:

$$\boldsymbol{R(\alpha) \le R_{emp}(\alpha) + f(h)}$$

$$f(h) = \sqrt{\frac{h(\log((2l/h) + 1)) - \log(\eta/4)}{l}}$$

$f(h)$ is referred to as the VC confidence, η is a confidence measure ($0 \le \eta \le 1$).

- Approach: **choose the machine that gives the least upper bound on actual risk**

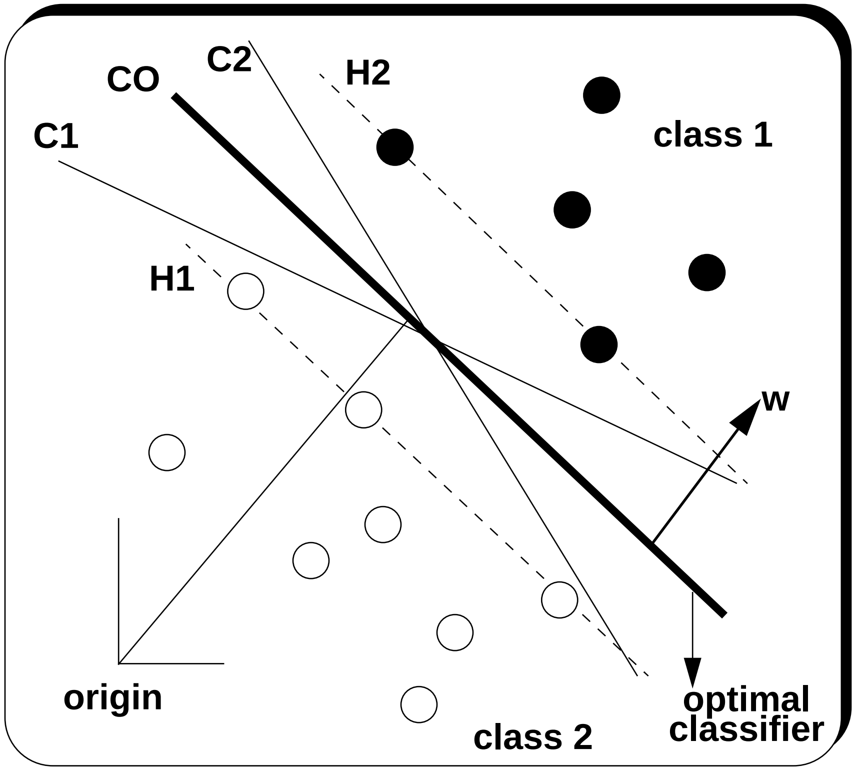


- The VC dimension, h is a measure of the capacity of the learning machine.

- Principle of structural risk minimization (SRM) (Vapnik, 1979) involves finding the subset of functions that minimizes the bound on the actual risk.

- Optimal hyperplane classifiers achieve zero empirical risk for linearly separable data.

SUPPORT VECTOR MACHINES



- Hyperplanes C0-C2 achieve perfect classification — zero empirical risk.
- C0 is optimal in terms of generalization.
- The data points that define the boundary are called support vectors.

Optimization (Separable Data)

- Hyperplane: $\boldsymbol{x} \cdot \boldsymbol{w} + b$
- Constraints:

$$y_i(\boldsymbol{x}_i \cdot \boldsymbol{w} + b) - 1 \geq 0 \qquad \forall i$$

The data points that satisfy the equality are called support vectors.

- Optimize:

$$L_P = \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_{i=1}^{N} \alpha_i y_i (\boldsymbol{x}_i \cdot \boldsymbol{w} + b) + \sum_{i=1}^{N} \alpha_i$$

- Minimization of this Lagrange functional minimizes risk criterion (maximizes margin).
- Final classifier:

$$f(\boldsymbol{x}) = \sum_{i=1}^{numSVs} \alpha_i y_i \boldsymbol{x}_i \cdot \boldsymbol{x} + b$$

SUPPORT VECTOR MACHINES

- Data for practical applications typically not separable using a hyperplane in the original input feature space

- Transform data to higher dimension where hyperplane classifier is sufficient to model decision surface

$$\Phi : \Re^n \rightarrow \Re^N$$

- Kernels used for this transformation

$$K(x_i, x_j) = \Phi(x_i) \cdot \Phi(x_j)$$

- Final classifier:

$$f(\mathbf{x}) = \sum_{i=1}^{numSVs} \alpha_i y_i K(\mathbf{x}, x_i) + b$$

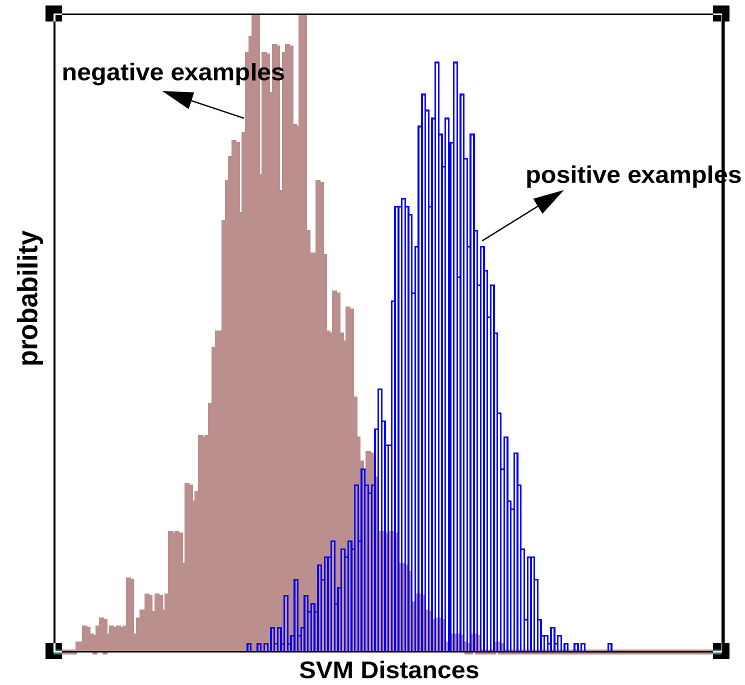
- Soft margin classifiers used in practice:

$$y_i(\mathbf{x}_i \cdot \mathbf{w} + b) \geq 1 - \xi_i \qquad \forall i$$

- SVMs do not generate likelihoods directly

- Posterior estimation required for speech

- Use a sigmoid function to map distances to posteriors:



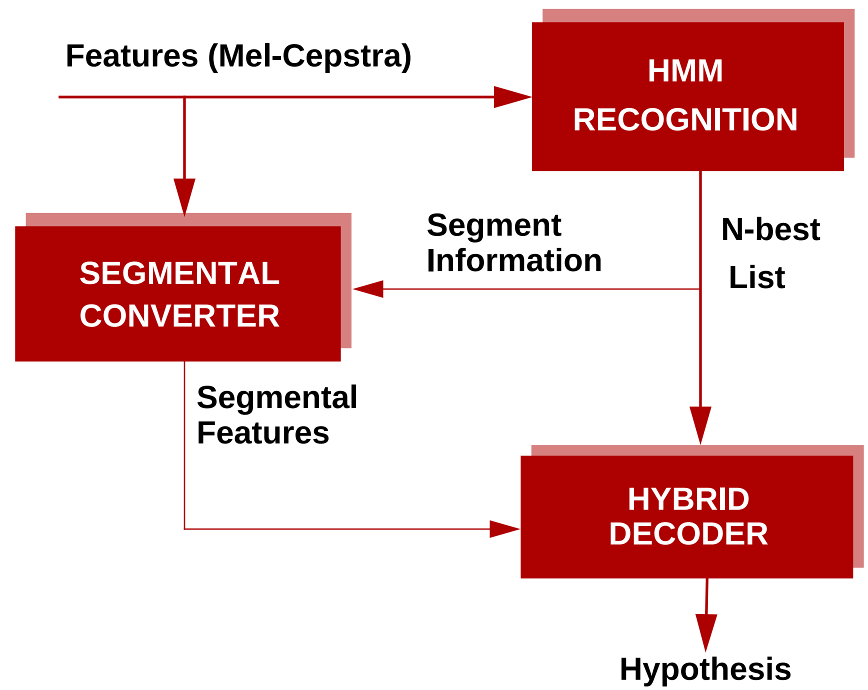
$$p(y = 1 / f) = \frac{1}{1 + \exp(Af + B)}$$

SUPPORT VECTOR MACHINES

- Experimental Results: Deterding Vowel (11 vowels spoken in “h*d” context)

Approach	Error Rate
K-Nearest Neighbor	44%
Gaussian Node Network	44%
SVM: Polynomial Kernels	49%
SVM: RBF Kernels	35%
Separable Mixture Models	30%
RVM: RBF Kernels	30%

- A Hybrid Speech Recognition Framework



- Experimental Results: Continuous Speech

Information Source		HMM		Hybrid	
Transcription	Segmentation	AD	SWB	AD	SWB
N-best	Hypothesis	11.9	41.6	11.0	40.6
N-best	N-best	12.0	42.3	11.8	42.1
N-best + Ref.	Reference	—	—	3.3	5.8
N-best + Ref.	N-best + Ref.	11.9	38.6	9.1	38.1

- Rescore N-best lists using phone classifiers
- Use a segmental modeling approach for phone classifiers
- 10.6% on AD task using hybrid system that combines HMM and SVM scores

BAYESIAN MODELING

- First level of inference:

$$P(\mathbf{w}|D, H_i) = \frac{P(D|\mathbf{w}, H_i)P(\mathbf{w}|H_i)}{P(D|H_i)}$$

$\mathbf{w}$: the set of adjustable parameters

D: data from which we make inferences

H_i: overall model

- Second level of inference:

$$\frac{P(H_1|D)}{P(H_2|D)} = \frac{P(D|H_1)P(H_1)}{P(D|H_2)P(H_2)}$$

if $P(H_1) = P(H_2)$, best model chosen by evaluating evidence $P(D|H_i)$.

- Evidence marginalized across model parameters:

$$P(D|H_i) = \int P(D|\mathbf{w},H_i)P(\mathbf{w}|H_i)d\mathbf{w}$$

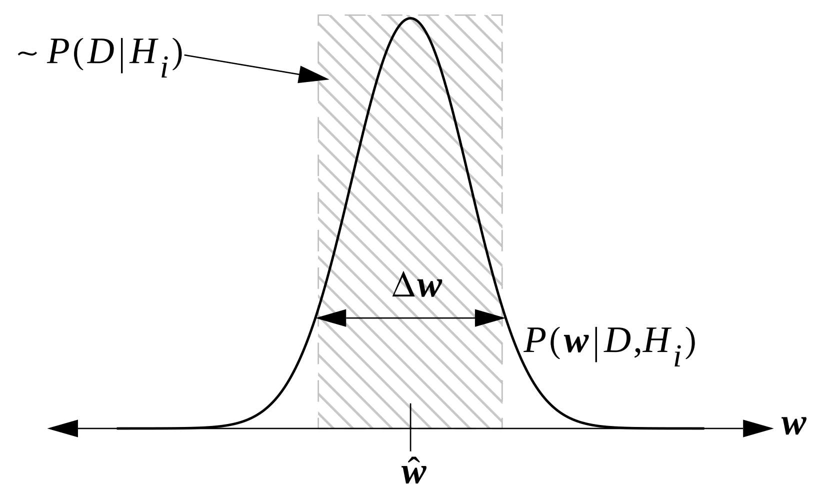
- It is impractical to compute this integral, so we need an approximation.

- Under the assumption that the posterior probability is Gaussian:

$$P(\mathbf{w}|D,H_i) \approx P(D|\mathbf{w},H_i)P(\mathbf{w}|H_i)$$

- The marginalization integral can be assumed to have a strong peak at the most probable value of the parameters, $\hat{\mathbf{w}}$.

- The evidence can then be approximated by multiplication of the height of the integrand and the width of the posterior, $\Delta\mathbf{w}$.



- Evidence approximation for a single model (Gaussian assumption)

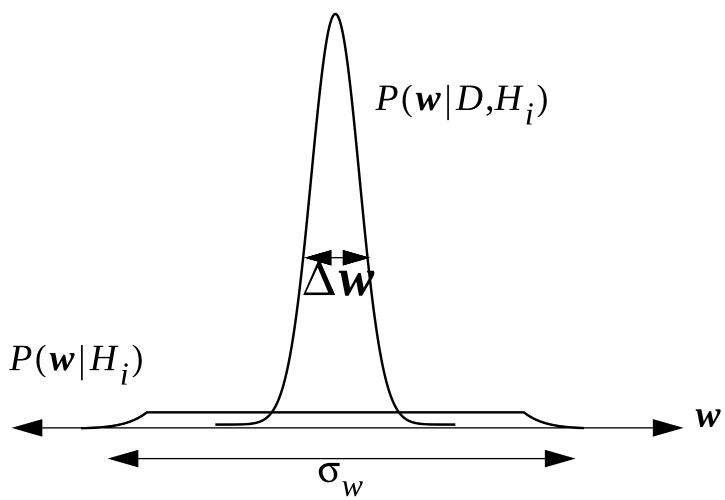
EVIDENCE FRAMEWORK

- The evidence is approximated by

$$P(D|H_i) \approx P(D|\hat{\mathbf{w}},H_i)P(\hat{\mathbf{w}}|H_i)\Delta\mathbf{w}$$

$P(D|\hat{\mathbf{w}},H_i)$ is the likelihood of the data given the best-fit parameter set

$P(\hat{\mathbf{w}}|H_i)\Delta\mathbf{w}$ is a penalty on the range of [0,1] which measures how well our posterior model fits our prior assumptions.



- The parameter's prior distribution and the posterior distribution width determine the model complexity

- The objective in training:

$$(\hat{\mathbf{w}}, \hat{\alpha}) = \underset{\mathbf{w},\,\alpha}{\operatorname{argmax}}\; p(\mathbf{w}, \alpha|\mathbf{t}, \mathbf{O})$$

- Using Bayes' rule:

$$p(\mathbf{w}, \alpha|\mathbf{t}, \mathbf{O}) = \frac{p(\mathbf{t}|\mathbf{w}, \alpha, \mathbf{O})\,p(\mathbf{w}, \alpha|\mathbf{O})}{p(\mathbf{t}|\mathbf{O})}$$

- A closed form solution to this maximization is not possible.
- An iterative approximation has been developed by MacKay that has complexity $O(N^3)$ and is based on Gaussian assumptions. Not feasible for large speech recognition tasks.
- This approach is similar to Minimum Description Length (MDL) and Bayesian Information Criterion (BIC).

RELEVANCE VECTOR MACHINES

Drawbacks of SVMs:

- Complexity scales linearly with the training data for nontrivial problems (prohibitive for large speech recognition tasks).
- Sparsity of the model should be explicit in the optimization of the model.
- Need a posterior probability, not distance.
- The sigmoid approximation tends to overestimate confidence (Tipping).

Relevance Vector Machines:

- A kernel-based learning technique.
- A Bayesian approach (MacKay) that incorporates an automatic relevance determination (ARD) prior over each model parameter.
- RVMs typically require an order of magnitude less parameters than SVMs, but require significantly more training time.

- As with SVMs, the RVMs are formed by defining a vector-to-scalar mapping:

$$y(\boldsymbol{o};\boldsymbol{w}) = w_o + \sum_{i=1}^{M} w_i \phi_i(\boldsymbol{o}) = \boldsymbol{w}^T \phi(\boldsymbol{o})$$

- RVMs take a Bayesian approach and explicitly define an ARD prior distribution over the weights:

$$p(\boldsymbol{w}|\alpha) = \prod_{i=0}^{N} N\left(w_i \middle| 0, \frac{1}{\alpha_i}\right) = \frac{1}{\sqrt{(2\pi)^{N+1}|\boldsymbol{A}^{-1}|}} e^{-\frac{1}{2}\boldsymbol{w}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{w}}$$

- To complete the Bayesian specification of the model, we use a non-informative (flat) prior for α_i.
- The likelihood of the training data set can be written as:

$$P(\boldsymbol{t}|\boldsymbol{w}, \boldsymbol{O}) = \prod_{n=1}^{N} \sigma_n^{t_n}(1-\sigma_n)^{1-t_n}$$

where $\sigma_n = \sigma\{y(\boldsymbol{o}_n;\boldsymbol{w})\}$.

SVM / RVM COMPARISON

Support Vector Machines

Data:

Class labels: {-1,+1}; “one vs. all”

Goal:

Find decision surface that maximizes the margin between two classes

Training:

Adjust parameters under constraint:

$$y_i(\boldsymbol{x}_i \cdot \boldsymbol{w} + b) \geq 1 - \xi_i \qquad \forall i$$

Optimize:

$$L_P = \frac{1}{2}\|\boldsymbol{w}\|^2 - \sum_{i=1}^{N} \alpha_i y_i (\boldsymbol{x}_i \cdot \boldsymbol{w} + b) + \sum_{i=1}^{N} \alpha_i$$

Training Complexity: $O(N^2)$

Classification: Threshold decoding (0.0)

Decoding:

- Rescoring N-best lists
- Segmental models

Relevance Vector Machines

Data:

Class labels: {0,1}; “one vs. all”

Goal: Learn posterior, $P(t|\boldsymbol{x})$.

Training:

$$y(\boldsymbol{x}) = \boldsymbol{w} \cdot \boldsymbol{x} + b$$

$$P(t|\boldsymbol{x}) = \frac{1}{1 + e^{-y(\boldsymbol{x})}}$$

$$p(\boldsymbol{w}|\alpha) = \prod_{i=0}^{N} N\left(w_i \middle| (\mu_i = 0), \frac{1}{\alpha_i}\right)$$

find: $\underset{\bar{\boldsymbol{w}}, \bar{\alpha}}{\operatorname{argmax}} P(\boldsymbol{w}, \alpha | [t], [x])$

iteratively find $\hat{\boldsymbol{w}}|\alpha$ then $\hat{\alpha}|\hat{\boldsymbol{w}}$.

Training Complexity: $O(N^3)$

Classification: Threshold decoding (0.5)

Decoding: Integrated likelihood computation

PROPOSAL

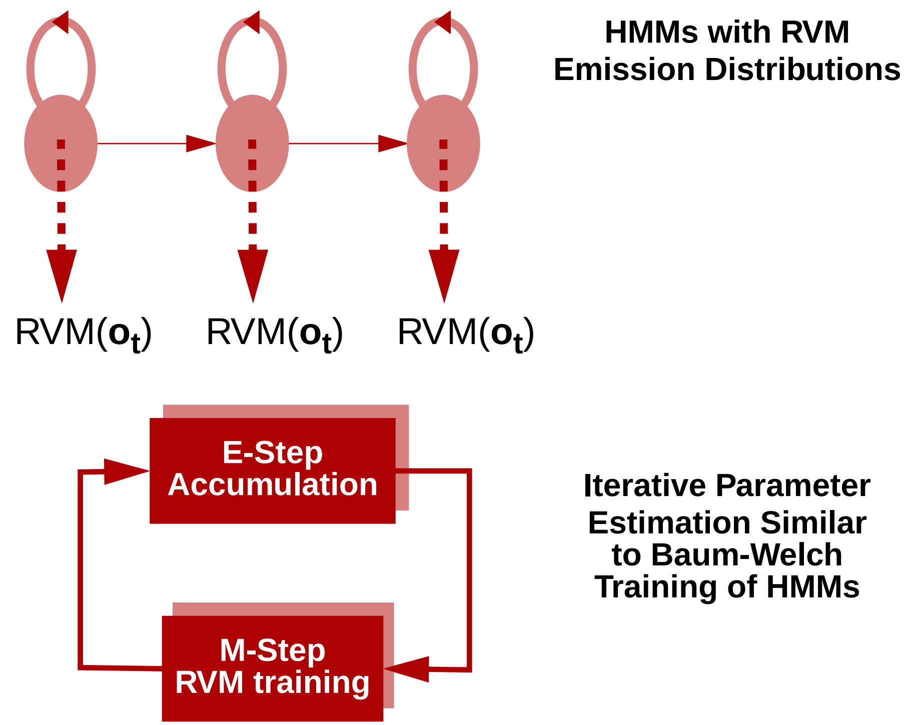
Current:

- Two-pass decoding methodology
- Ad hoc method for determining optimal segmentation
- Ad hoc method for determining posterior probability
- Lacks iterative training

Proposed:

- Integrated HMM/RVM solution
- Frame-based modeling: eliminates need for segmental models
- RVM is naturally probabilistic: eliminates need for sigmoid posterior fit

Proposed Architecture:



- Convergence properties and efficient training methods are critical.
- Bootstrapping or incremental training
- Available as part of the ISIP speech recognition toolkit.

RESEARCH PLAN

Practical optimization methods

- Currently $O(N^2)$ in memory and $O(N^3)$ in time - prohibitive for large data sets.
- Explore methods for incremental learning: Active learning (MacKay) or decomposition (Tipping)

Integrated, iterative HMM/RVM training

- RVM replaces Gaussian
- E-M style training paradigm
- Need to address issues such as convergence and parameter tying

Integrated HMM/RVM decoder

- Single-pass decoding
- Parameter tuning necessary

Experimental Progression

Task	System	Data	Date
Static Classification	Set of 1-vs-All classifiers	Deterding Vowel	March
Pilot ASR	Hybrid HMM/RVM (same as SVM) 2000 training	Alphadigit	March
Practical Optimization	Set of 1-vs-All classifiers	Deterding Vowel	April
	Hybrid HMM/RVM Full training	Alphadigit	April
HMM/RVM Training and Decoding	Frame-based models Full training. First with single frame feature vectors, then with extended feature set	TIDigits 13000 train, 13000 test	June
		Alphadigits 60k train 3300 test	July
		SWB 114k train 2400 test	Aug

PRELIMINARY EXPERIMENTS

- Experimental Results: Deterding Vowel (11 vowels spoken in “h*d” context)

Approach	Error Rate
K-Nearest Neighbor	44%
Gaussian Node Network	44%
SVM: Polynomial Kernels	49%
SVM: RBF Kernels	35%
Separable Mixture Models	30%
RVM: RBF Kernels	30%

Approach	Avg. Parameter Count
SVM: RBF Kernels	83 SVs
RVM: RBF Kernels	13 RVs

- RVMs yield superior sparsity with comparable generalization.

- Experimental Results: OGI Alphadigits (telephone bandwidth letters and numbers)

Approach	Error Rate	Avg. Parameter Count	Training Time	Testing Time
SVM	16.4%	257 SVs	1/2 hour	30 mins
RVM	16.2%	12 RVs	1 month	1 min

- Hybrid RVM system is mirror of hybrid SVM system (still has segmentation problem).

- Reduced training set size (2000 examples per phone class).

- RVM yields a large reduction in parameter count — translates to large efficiency boost for decoder.

- Computational cost mainly in training, but is still prohibitive for large data sets.

DISSERTATION CONTRIBUTIONS

What I will do:

- Build the first kernel-machine-based single-pass decoder.
- Make the first application of RVMs to speech recognition.
- Overcome the problems in the hybrid HMM/SVM system: fully probabilistic models; extremely sparse models; models trained iteratively and in an integrated manner.
- Package the software and documentation into our public-domain recognizer.

What I will not do:

- Build a comparison hybrid ANN system.
- Implement all of current speech recognition in terms of RVMs. e.g. no RVM adaptation, but this is ripe for future work!

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