

ANALYTIC SIGNAL PROCESSING

BY

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## TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENT . . . . .	iii
LIST OF TABLES . . . . .	vi
LIST OF FIGURES . . . . .	vii
ABSTRACT . . . . .	x
CHAPTER	
I. INTRODUCTION . . . . .	1
II. ANALYTIC SIGNALS . . . . .	8
Introduction . . . . .	8
Analog Analytic Signals . . . . .	8
Discrete Analytic Signals . . . . .	11
III. ANALYSIS OF WINDOWED ANALOG SIGNALS . . . . .	17
Introduction . . . . .	17
The Spectrum of a Real Sinewave . . . . .	17
The Spectrum of the Sum of Two Real Sinewaves . . . . .	21
Analysis of a Complex Sinewave . . . . .	23
Analysis of the Sum of Two Complex Sinewaves . . . . .	24
Spectral Stationarity of an Analytic Signal . . . . .	26
IV. ANALYSIS OF FINITE LENGTH DISCRETE SEQUENCES . . . . .	30
Introduction . . . . .	30
The Discrete Spectrum . . . . .	30
The Discrete Spectrum of Multiple Sinewaves . . . . .	33
Time Domain Windowing . . . . .	37
V. LINEAR PREDICTION USING AN ANALYTIC SIGNAL . . . . .	39
Introduction . . . . .	39
The Complex Linear Predictor . . . . .	39
Derivation of the Step-Down Procedure . . . . .	47
Phase Invariance of the Analytic Signal-Based Linear Predictor . . . . .	51
Downsampling the Analytic Signal . . . . .	54
Frequency Estimation of a Single Sinewave . . . . .	55
Frequency Estimates of Multiple Sinewaves . . . . .	58

CHAPTER		Page
VI.	LEAST SQUARES MULTI-PULSE LINEAR PREDICTIVE CODING . . . . .	63
	Introduction . . . . .	63
	Least Squares Identification of Speech	
	Using a Non-Ideal Driving Function . . . . .	64
	A Sub-Optimal Algorithm for Joint Process	
	Estimation . . . . .	71
	Multi-Pulse Linear Predictive Coding . . . . .	80
VII.	ANALYTIC MULTI-PULSE LINEAR PREDICTIVE CODING . . . . .	84
	Introduction . . . . .	84
	Pulse Location for a Complex Signal . . . . .	84
	Complex Parameter Coding . . . . .	90
	Complex to Real Parameter Transformations . . . . .	99
	Complex Pulse Coding . . . . .	104
	A 9.6 kbits/s AMPLPC System . . . . .	105
	A Comparison of the Performance of	
	AMPLPC to MPLPC . . . . .	107
VIII.	SUMMARY . . . . .	110
IX.	FIGURES . . . . .	113
	REFERENCES . . . . .	161

LIST OF TABLES

Table		Page
1.	Complex Reflection Coefficient Bit Allocations	97

## LIST OF FIGURES

Figure		Page
1.	Reconstruction of a Real Signal from a Downsampled Analytic Signal . . . . .	114
2.	Aliasing in the Spectrum of a Single Real Windowed Sinewave . . . . .	115
3.	Composite Magnitude Spectrum of a Single Real Windowed Sinewave . . . . .	116
4.	Dependence of the Magnitude Spectrum on Window Length . . . . .	117
5.	Dependence of the Magnitude Spectrum on Phase . . . . .	118
6.	Aliasing in the Spectrum of Two Sinewaves . . .	119
7.	Composite Spectrum of Two Sinewaves . . . . .	120
8.	Composite Spectrum of a Complex Sinewave . . .	121
9.	Aliasing in the Spectrum of Two Complex Sinewaves . . . . .	122
10.	Composite Spectrum of Two Complex Sinewaves . .	123
11.	Phase Rotation of a Real Signal . . . . .	124
12.	Dependence of the Discrete Magnitude Spectrum of a Real Sinewave on Window Length . . . . .	125
13.	Dependence of the Discrete Magnitude Spectrum of a Real Sinewave on Phase . . . . .	126
14.	Invariance of the Discrete Spectrum of a Complex Sinewave to Window Length . . . . .	127
15.	Complex Lattice Filter Analyzer . . . . .	128
16.	Maximum Entropy Spectra of a Real Sinewave . .	129
17.	Maximum Entropy Spectra of a Complex Sinewave .	130
18.	Distribution of Frequency Estimates for Two Real Sinewaves in Noise ( $f_1 = 367$ Hz, $f_2 = 859$ Hz) . . . . .	131

Figure		Page
19.	Distribution of Frequency Estimates for Two Complex Sinewaves in Noise ( $f_1 = 367$ Hz, $f_2 = 859$ Hz) . . . . .	132
20.	Dependence of the Variance of the Frequency Estimates on Window Length (SNR = 20 dB) . . .	133
21.	Dependence of the Variance of the Frequency Estimates on Window Length (SNR = 40 dB) . . .	134
22.	Dependence of the Variance of the Frequency Estimates on Window Length (SNR = 60 dB) . . .	135
23.	Dependence of the Variance of the Frequency Estimates on Signal to Noise Ratio (N = 20 points) . . . . .	136
24.	Dependence of the Variance of the Frequency Estimates on Signal to Noise Ratio (N = 40 points) . . . . .	137
25.	Dependence of the Variance of the Frequency Estimates on Signal to Noise Ratio (N = 80 points) . . . . .	138
26.	Dependence of the Variance of the Frequency Estimates on Phase (SNR = 20 dB) . . . . .	139
27.	Dependence of the Variance of the Frequency Estimates on Phase (SNR = 40 dB) . . . . .	140
28.	Dependence of the Variance of the Frequency Estimates on Phase (SNR = 60 dB) . . . . .	141
29.	Dependence of the Variance on Window Length for the Autocorrelation Method (SNR = 20 dB) . . . . .	142
30.	Dependence of the Variance on Window Length for the Autocorrelation Method (SNR = 40 dB) . . . . .	143
31.	Dependence of the Variance on Window Length for the Autocorrelation Method (SNR = 60 dB) . . . . .	144
32.	Irregularly Spaced Pitch Pulses at the Onset of a Voiced Interval . . . . .	145
33.	Discontinuous Pitch Period Behavior . . . . .	146
34.	Typical Secondary Excitation Pulses . . . . .	147



Figure		Page
35.	A Female Speech Signal and Its Residual . . . . .	148
36.	Comparison of LPC and LSMPLPC Impulse Responses	149
37.	Comparison of LPC and LSMPLPC Spectra . . . . .	150
38.	A Male Speech Signal and Its Residual . . . . .	151
39.	Comparison of LPC and LSMPLPC Impulse Responses	152
40.	Comparison of LPC and LSMPLPC Spectra . . . . .	153
41.	Distributions of the First Four Complex Reflection Coefficients . . . . .	154
42.	Distributions of the Second Four Complex Reflection Coefficients . . . . .	155
43.	Magnitude/Phase Distributions of the First Four Complex Reflection Coefficients . . . . .	156
44.	Magnitude/Phase Distributions of the Second Four Complex Reflection Coefficients . . . . .	157
45.	Two-Dimensional Quantizers . . . . .	158
46.	The AMPLPC System . . . . .	159
47.	Frequency Response of a Discrete Hilbert Transformer . . . . .	160

## ABSTRACT

High resolution spectral estimation algorithms traditionally are constrained to process a finite segment of data, assuming the data to be zero outside the analysis interval. These assumptions ultimately limit the resolution which can be achieved by these estimators. Analytic signals provide an alternate signal representation whereby long-term phase information can be incorporated into the analysis data. Analytic signal-based spectral estimators are shown to achieve higher resolution than their real signal counterparts, due to the phase-invariance property of an analytic signal. This is demonstrated in both the analog and discrete domains. The linear predictive estimates of stationary signals in additive Gaussian noise, using a complex linear predictor, are shown to be more consistent than those obtained using a comparable real linear predictor. Experimental results are included for two analysis algorithms: the Burg algorithm and the autocorrelation method.

The analytic signal representation has been applied to data compression techniques for speech encoding. Multi-pulse linear predictive coding (MPLPC) is developed as a sub-optimal two-stage system identification algorithm. Several complex parameter coding algorithms are presented as sub-optimal solutions to a two-dimensional quantization problem. Two techniques to transform complex predictor coefficients to equivalent real predictor coefficients are

included as alternate coding algorithms. An analytic multi-pulse linear predictive coding system (AMPLPC) has been designed which provides improved speech quality over its real signal counterpart at rates ranging from 9.6 kbits/s to 14.4 kbits/s.