

# Automatic Scaling Range Selection for Long-range Dependent Network Traffic

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**Abstract**—In this paper, we present an adaptive search algorithm to automatically select the scaling range in the wavelet-based Hurst parameter estimation method. This algorithm is recursive and adaptive in nature, and it can select a scaling range consistent with human visual selection. In addition, it can be easily extended to automatically find the (approximately) linear regions of any curve. We tested our algorithm on 13 NLANR network traffic traces. The results show that our algorithm works well for the cases of monofractal traffic.

## I. INTRODUCTION

It is believed [1] that the high variability in Internet traffic is due to the *long-range dependence* (LRD) property of the traffic processes. In general, a (weakly) stationary discrete-time real-valued stochastic process  $Y = \{Y_n, n = 0, 1, 2, \dots\}$  with mean  $\mu = E[Y_n]$  and variance  $\sigma^2 = E[(Y_n - \mu)^2] < \infty$  exhibits LRD if  $\sum_{k=1}^{\infty} r(k) = \infty$ , where  $r(k)$  measures the correlation between samples of  $Y$  separated by  $k$  units of time. If  $\sum_{k=1}^{\infty} r(k) < \infty$ , then  $Y$  is said to exhibit *short-range dependence* (SRD).

Common traffic models with LRD are based on self-similar processes. In traffic modeling, the term self-similarity is usually used to refer to the *asymptotically-second order self-similar* processes [1]. By definition, asymptotically second-order self-similarity implies LRD and vice versa [1].

The parameter  $H$  is called the *Hurst parameter*. For general self-similar processes, it measures the degree of “self-similarity.” For random processes suitable for modeling network traffic, the Hurst parameter is basically a measure of the speed of decay of the tail of the autocorrelation function. If  $0.5 < H < 1$ , then the process is LRD, and if  $0 < H \leq 0.5$ , then it is SRD. Hence,  $H$  is widely used to capture the intensity of long-range dependence in a traffic process. The closer  $H$  is to 1 the more long-range dependent the traffic is, and vice versa.

There are several methods for estimating  $H$  from a traffic trace. One of the most widely used is based on wavelets [2]. Given a traffic trace  $Y_n$ ,  $H$  can be estimated as follows:

- First, for each *scale*  $j$  and *position*  $k$ , compute the wavelet coefficients:  $d(j, k) = \langle Y_n, \Psi_{j,k}(n) \rangle = \sum_{n=1}^{\infty} Y_n \Psi_{j,k}(n)$  where  $\Psi_{j,k}(n) = 2^{-j/2} \Psi_0(2^{-j}n - k)$  and  $\Psi_0$  is the (Daubechies) mother wavelet [2].
- Then, compute the wavelet energy  $\mu_j$  for each scale  $j$ :  $\mu_j = \frac{1}{N_j} \sum_{k=1}^{N_j} d^2(j, k)$  where  $N_j$  is the total number of wavelet coefficients at scale  $j$ .
- Next, make a plot of  $\log_2(\mu_j)$  versus scale  $j$  and apply linear regression over the curve region that *looks* linear. Compute the slope  $\alpha$ .

- Finally, estimate the Hurst parameter as  $\hat{H} = \frac{\alpha+1}{2}$ .

The scaling behavior of internet traffic is not strictly self-similar, but rather more complex [5]. By using the wavelet method above, we will usually see that the  $\log_2(\mu_j)$  versus  $j$  curve is not strictly linear (for example, see Figures 1-3). Below a certain cut-off scale  $j_1$ , the scaling behavior is not self-similar. (It is believed that internet traffic below the lower cut-off scale  $j_1$  exhibits multi-fractal scaling behavior.) Above a certain higher cut-off scale  $j_2$ , there will be few transformed wavelet coefficients, and the estimation of the Hurst parameter will be quite noisy [3]. When estimating the Hurst parameter, it is better to discard data in the fine scales less than  $j_1$  and coarse scales greater than  $j_2$ .

So far, to estimate the Hurst parameter using the wavelet-based method, a visual inspection of the  $\log_2(\mu_j)$  vs.  $j$  plot is necessary to identify the linear trend region of the curve (see step 3 above). This becomes a problem because it is not objective. Furthermore, there are situations in which visual inspection is not possible such as real-time automatic Hurst parameter estimation. This problem is very important in the process of determining the Hurst parameter by the wavelet method.

In this paper, we present a robust, adaptive and recursive algorithm that automatically searches and determines the linear region of the curve. To the best of our knowledge, there is no other such algorithm proposed in the literature. The algorithm recursively finds the scales  $j_1$  and  $j_2$  for which the curve over the range  $[j_1, j_2]$  seems linear. This algorithm is based on the greedy algorithm principle [6]. It first finds the local optimum subsolution of the starting  $[j_1, j_2]$  interval. It then expands the interval (see next section) and finds again the local optimum subsolution. This process of the algorithm continues until this series of local optimum subsolutions converges to the global optimum solution. The algorithm is simple and fast with complexity  $O(n)$ , and thus it can be incorporated into the present wavelet-based method for real-time Hurst parameter estimation of monofractal network traffic. We tested our algorithm with 13 long traffic traces from the Auckland-II, Auckland-IV, NZIX-II, Bell Labs-I, and Abilene-I data archives [7].

The rest of this paper is organized as follows. Section II presents our proposed algorithm. Section III shows some scaling range results on real traffic traces using our algorithm. Our conclusions are presented in section IV.

## II. THE ALGORITHM

To determine the linear region of a curve, a natural approach is to use linear regression for modeling the observational data. In deriving such a model, we measure how closely the model characterizes the observational (experimental) data. Rather than the usual residual (the sum of the squares of the deviations), our approach uses Theil's inequality coefficient  $U$  [4]. Since it is normalized,  $U$  is a data-independent measure of the goodness-of-fit. It is defined as follows:

$$U = \frac{\sqrt{(1/N) \sum_{j=1}^N (Y_j^s - Y_j^a)^2}}{\sqrt{(1/N) \sum_{j=1}^N (Y_j^s)^2} + \sqrt{(1/N) \sum_{j=1}^N (Y_j^a)^2}} \quad (1)$$

where  $Y_j^s$  denotes the model derived values,  $Y_j^a$  denotes the observational values and  $N$  denotes the total number of observational data points. The numerator is the square root of mean square error, and the denominator is used to normalize  $U$  in the range of 0 and 1. If  $U$  is 0, then the model's simulation results match exactly the observational data. As  $U$  approaches 1, the accuracy of the model worsens.

The algorithm searches and determines the linear region of the curve. Given a  $\log_2(\mu_j)$  vs.  $j$  curve for a traffic trace  $Y_n$ , we begin by choosing  $j_1 < j_2$  in the middle region<sup>1</sup> of the curve. Then, while the Theil's parameter for the range  $[j_1, j_2]$  is below some given threshold  $\gamma_0$ , we expand the range either left to  $[j_1 - 1, j_2]$  or right to  $[j_1, j_2 + 1]$ , always choosing the expansion with the smaller value of Theil's parameter. So as long as this process continues, the linear range  $[j_1, j_2]$  will stretch to its optimal width.

The following shows the algorithm in detail:

- 1) Set the initial values of  $j_1$  and  $j_2$  as follows:  $j_1 = \frac{M}{2} - 1$ ,  $j_2 = \frac{M}{2} + 1$ , where  $M$  is the maximum wavelet scale. Let  $L$  be the length of  $Y_n$ , then  $M = \log_2(L)$ .
- 2) Using the least square method, fit a linear line between  $j_1$  and  $j_2$ :  $y(j) = kj + b + \varepsilon_j$ . where  $y(j) = \log_2(\mu_j)$  and  $\varepsilon_j$  is the error associated with point  $j$ . Letting  $N = j_2 - j_1 + 1$ , the slope  $k$  and constant  $b$  are computed as follows:
$$k = \frac{N \sum_{j=j_1}^{j_2} j y(j) - \sum_{j=j_1}^{j_2} j \sum_{j=j_1}^{j_2} y(j)}{N \sum_{j=j_1}^{j_2} j^2 - (\sum_{j=j_1}^{j_2} j)^2}$$

$$b = \frac{\sum_{j=j_1}^{j_2} y(j)}{N} - k \frac{\sum_{j=j_1}^{j_2} j}{N}$$
- 3) Next, compute Theil's  $U$  parameter using (1) and data between  $j_1$  and  $j_2$ . Note that here  $Y_j^s = kj + b$ ,  $Y_j^a = y(j)$ ,  $j = j_1, \dots, j_2$ :  $\delta(j_1, j_2) = U$
- 4) If  $\delta(j_1, j_2) \leq \gamma_0$  then compute  $\delta(j_1 - 1, j_2)$  and  $\delta(j_1, j_2 + 1)$ .
  - a) If  $\delta(j_1 - 1, j_2) < \delta(j_1, j_2 + 1)$  and  $\delta(j_1 - 1, j_2) < \gamma_0$  then set  $j_1 \leftarrow j_1 - 1$  and go back to Step 2.
  - b) If  $\delta(j_1, j_2 + 1) < \delta(j_1 - 1, j_2)$  and  $\delta(j_1, j_2 + 1) < \gamma_0$  then set  $j_2 \leftarrow j_2 + 1$  and go back to Step 2.
- 5) else Stop.

Obviously, the value of the residual error threshold  $\gamma_0$  determines the *goodness-of-fit*. We recommend using the threshold value  $\gamma_0=0.015$ . On one hand, Table II shows that this threshold produces linear ranges that closely match the ranges obtained by visual inspection. On the other hand, such  $\gamma_0$  is close enough to zero to yield a good estimate of the Hurst parameter.

<sup>1</sup>All traffic traces analyzed for this paper exhibit monofractal behavior that spans the middle region of the wavelet curve. Therefore, for simplicity, we let the algorithm to start progressing from the middle region of the curve. However, this is not a necessary condition of the algorithm.

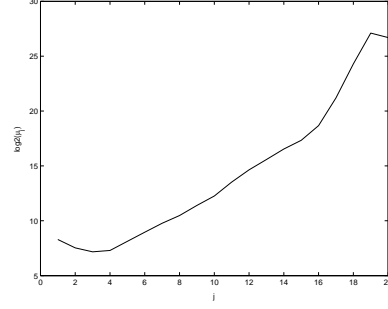


Fig. 1.  $\log_2(\mu_j)$  vs. scale  $j$  for the Auckland-IV traffic trace 20010301-310-0

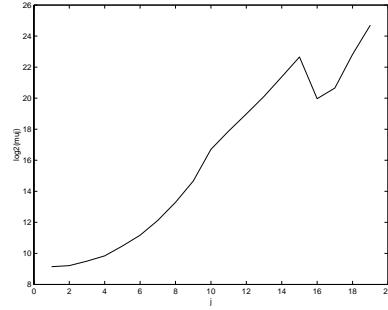


Fig. 2.  $\log_2(\mu_j)$  versus scale  $j$  for the Bell Labs-I traffic trace 20020519-151927.

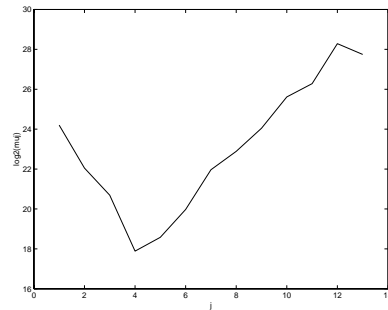


Fig. 3.  $\log_2(\mu_j)$  versus scale  $j$  for the Abilene-I traffic trace IPLS-KSCY-20020814-090000-1.

We implemented the above algorithm in Matlab and obtained *automatically* the linear trend ranges  $[j_1, j_2]$  for the 13 long traffic traces.

To illustrate how the algorithm works, we take traffic trace 20010301-310-0 as an example to demonstrate how the algorithm gradually searches for the optimal linear scaling range. Table I shows detailed search results step by step.

TABLE I  
STEP BY STEP ILLUSTRATION OF LINEAR RANGE SELECTION BY ADAPTIVE SEARCH ALGORITHM FOR THE CURVE IN FIGURE 1.

number of points(N)	[j <sub>1</sub> ,j <sub>2</sub> ]	stretch direction	theil U
3	[9,11]		0.0039
4	[9,12]	right	0.0035
5	[9,13]	right	0.0032
6	[9,14]	right	0.0033
7	[8,14]	left	0.0036
8	[8,15]	right	0.0042
9	[8,16]	right	0.0041
10	[7,16]	left	0.0052
11	[6,16]	left	0.0064
12	[5,16]	left	0.0075
13	[4,16]	left	0.0084

In Table II, the linear scaling range obtained using visual inspection and the adaptive search algorithm are compared. Clearly, the algorithmically obtained linear regions for the

TABLE II  
COMPARISON BETWEEN ALGORITHMICALLY OBTAINED LINEAR TREND REGIONS AND VISUAL INSPECTION FOR THE AUCKLAND TRAFFIC TRACES.

Traffic Trace	by inspection	by algorithm	$\hat{H}$
19991129-134258-0	[4,20]	[3,20]	0.813
19991129-134258-1	[6,20]	[7,20]	0.968
19991201-192548-0	[3,19]	[3,19]	0.890
nzix-II	[4,15]	[4,16]	0.979
20010220-226-0	[3,15]	[4,14]	0.836
20010220-226-1	[4,15]	[4,15]	0.891
20010301-310-0	[4,16]	[4,16]	0.919
20010301-310-1	[4,16]	[4,16]	0.903
20020519-000000	[5,10]	[2,9]	0.564
20020519-151927	[2,15]	[4,15]	0.964
IPLS-CLEV-20020814-0	[4,12]	[5,12]	1.299
IPLS-CLEV-20020814-1	[4,12]	[4,12]	0.965
IPLS-KSCY-20020814-1	[4,12]	[4,12]	1.121

$\log_2(\mu_j)$  vs.  $j$  curves are consistent with ones obtained from visual inspection. To estimate the Hurst parameter we used the Matlab routine *LDestimate.m* which is available in [8]. We observe from Table II that the estimated Hurst parameter values for two of the IPLS traffic traces are greater than one. The explanation of this is beyond of the scope of this paper. However, the reader should note that there are well-defined self-similar processes with stationary increments, infinite second moments, and  $H \geq 1$  [9].

In this paper, we presented a robust and adaptive algorithm that can be incorporated in the wavelet-based method of estimating the Hurst parameter for monofractal traffic traces, that is, traffic traces that exhibit monofractal behavior over a wide range of time scales. It provides a systematic and objective way to determine the linear trend region instead of subjective human visual inspection. It is based on the greedy algorithm principle and uses Theil's inequality measure. Our algorithm automatically searches and determines the linear region of the  $\log_2(\mu_j)$  vs.  $j$  curve. That is, the algorithm recursively finds the scales  $j_1$  and  $j_2$  for which the curve over  $[j_1, j_2]$  looks linear.

This algorithm is an improvement to the process of estimating the Hurst parameter in the wavelet domain. It can be easily integrated with various traffic control schemes that require *real-time* Hurst parameter estimations. It is known that any wavelet-based curve of a truly monofractal process exhibits a linear behavior over a region that spans the middle scale values [1], [2]. Thus, we configured our algorithm to start processing from the middle region of the observational curve. When we tested our algorithm with 13 long NLANR traffic traces, we obtained satisfactory results. These traffic traces exhibit monofractal behavior that spans the middle region of the wavelet curve, otherwise the processes would have been classified as multifractal.

The algorithm can be easily generalized in respect to the starting point, as subject of future work. It can be easily modified such that, for example, the initial values of  $j_1$  and  $j_2$  are set to be the two end scale values (i.e., first and last). In this case, the algorithm should be extended to automatically determine all the linear regions of a curve and then have the intelligence to determine the linear zone that corresponds to the monofractal behavior of the traffic process.

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