SRLF

Request Date: 06-JUN-2012

Expiration Date: 11-JUN-2012

ILL Number: 5253132

Call Number:

Format: Article Printed

Author: ASSP Workshop on Spectrum Estimation and Modeling (3rd : 1986 : Northeastern University)

Title: Third ASSP Workshop on Spectrum Estimation and Modeling : November 17-18, 1986, Northeastern University, ~~Boston,~~ Massachusetts. p.172-177 KC

Article Title: J. Picone: Spectrum Estimation Using an Analytic Signal Representation

Part Pub. Date: 1986

Pub. Place: New York, NY : Institute of Electrical and Electronics Engineers ; Piscataway, NJ : Additional copies may be obtained from IEEE Service Center, c1986.

Borrower: UCLA Libraries (Parent Location)

Patron Name: Picone, Joseph

Patron e-mail:

Service Level: Normal - Local Search

Delivery Method:

Request Notes: We are a SHARES library. FAX/ARIEL:215-204-0753 ARIEL: 155.247.55.82 EMAIL:teuill@temple.edu

Need By:

Verification Source: <TN:174465><ODYSSEY:206.107.42.224/PALEY> OCLC

Supplier Reference:

Supplier Reference: ILLNUM:91592727

Local request number: ILLNUM:91592727

Owned By: UCLA Young Research Library - ILL

Printed Date: 06-JUN-2012

TGQ or OCLC #: 5253131

ID: ULA0

ISBN/ISSN:

ARIEL SHARES

Address: Paley Library ILL/Temple University/1210 W. Berks St./Philadelphia, PA 19122

Service Type: Copy non returnable

Max Cost: USD30

Payment Type: IFM

Copyright Info: US:CCL

Requester Symbol: OCLC:TEU

Return To: UCLA Young Research Library-ILL
280 Charles E. Young Dr.
Los Angeles, CA,
90095-1575

SPECTRUM ESTIMATION USING AN ANALYTIC SIGNAL REPRESENTATION [1,2]

Joseph Picone
Dimitrios P. Prezas
Walter T. Hartwell
AT&T Bell Laboratories
Naperville, Illinois 60566

Joseph L. LoCicero
Department of Electrical And Computer Engineering
Illinois Institute of Technology
Chicago, Illinois 60616

ABSTRACT

High resolution spectrum estimation algorithms traditionally are constrained to process a finite amount of data, assuming the data to be zero outside the analysis interval. These assumptions ultimately limit the resolution that can be achieved by these estimators. Analytic signals provide an alternate signal representation whereby long-term phase information can be incorporated into the analysis data. Analytic signal-based estimators are shown to achieve higher resolution than their real signal counterparts, due to the phase-invariance property of an analytic signal. The linear predictive estimates of stationary signals in additive white Gaussian noise, using a complex linear predictor, are shown to be more consistent than those obtained using a comparable real linear predictor.

INTRODUCTION

Since the advent of digital signal processing in the late 1950s, the goal of producing the digital equivalent of the analog spectrum analyzer has spawned literally hundreds of digital spectral estimation techniques. Some of these fall in the category of "high resolution spectral estimators" while others are simply derivatives of the analog Fourier transform. These techniques and their relative shortcomings have been well documented in the literature [1]. The common thread among all the digital techniques is that they involve the estimation of the spectrum of a sampled, bandlimited, stationary time series from a finite number of samples of that process.

In this paper, the conventional approach of processing a single set of samples of a one-dimensional signal is discarded in favor of operating on an analytic representation of this signal. Throughout this work, it is assumed that a continuous stream of data is available to form the analytic signal, as is typically the case in such applications as speech processing. The analytic signal is shown to incorporate the long-term phase information of the signal into the analysis interval. Surprisingly, the computational burden required by the complex, or two-dimensional, sequence is comparable to its real, or one-dimensional, equivalent.

Analytic signal processing of one-dimensional signals such as speech is not an entirely new idea. Hartwell [2] proposed the analytic signal as a basis for conventional FFT analysis. In [3], some basic issues of frequency estimation and linear prediction are explored. The notions of the Hilbert transform envelope have been used in pitch detection [4]. In [5], analytic signals are applied to speech processing.

This paper presents a unified view of conventional time domain window theory and its relation to analytic signal processing. In the second section of this paper, the phase invariance property of analytic signal is derived in the context of windowed analog signals. In the third section, the phase invariance property is extended to linear predictive spectral estimation. In the fourth section, the performance of the analytic signal based estimator is compared to an equivalent real signal estimator for the problem of two sinewaves in additive noise.

ANALYSIS OF WINDOWED ANALOG SIGNALS

The inherent advantages in using an analytic signal representation result from the constraint of a finite length analysis interval. This is best illustrated by considering the effects of a rectangular window upon an analog signal consisting of a sum of two sinewaves of arbitrary frequencies and phases. Suppose the signal of interest, s(t), is defined as

$$s(t) = \cos(\omega_1 t + \phi + \phi_1) + \cos(\omega_2 t + \phi + \phi_2) \quad (1)$$

The phase angle, ϕ, common to both sinewaves, will denote the general phase of the signal with respect to the window. The spectrum of the rectangularly windowed version of this signal, $S_W(\omega)$, is given by

$$\begin{aligned} S_W(\omega) = {} & \pi\tau\{e^{j(\phi+\phi_1)}\text{Sinc}(x-x_1) \\ & + e^{-j(\phi+\phi_1)}\text{Sinc}(x+x_1)\} \\ & + \pi\tau\{e^{j(\phi+\phi_2)}\text{Sinc}(x-x_2) \\ & + e^{-j(\phi+\phi_2)}\text{Sinc}(x+x_2)\}, \end{aligned} \quad (2)$$

1. This research was supported in full by the Advanced Switching Services Laboratory, AT&T Bell Laboratories, Naperville, Illinois.

2. This work was submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering in the Graduate School of IIT, Chicago, Illinois.

where $x = \omega\tau/2$, $x_1 = \omega_1\tau/2$, and $x_2 = \omega_2\tau/2$. The peak in the spectrum for each sinewave will be shifted due to aliasing. There are two contributing factors to this peak shifting. First, there is the usual aliasing between positive and negative frequency components of the signal. Second, there is additional "in-band" aliasing between the two positive frequency components, and the two negative frequency components, more traditionally termed leakage [6]. Note that both types of aliasing are a function of the phase angle, ϕ. The spectral errors associated with real signals arise from the windowing process itself, and ultimately limit the resolution which can be achieved with a particular window.

The obvious remedy to completely eliminate the aliasing between positive and negative frequency components is to remove the negative frequency components via an analytic signal representation [5]. Let $s_a(t)$ be an analytic signal that satisfies the Dirichlet conditions [5]. The window, w(t), can represent any type of real or complex window function, provided it is of finite duration, and its Fourier transform exists; in general, let

$$w(t) = f(t), \quad -\tau/2 \le t \le \tau/2, \qquad = 0, \quad \text{elsewhere}. \tag{3}$$

Let $s_{ap}(t)$ be a phase rotated version of $s_a(t)$, where phase rotated implies some fixed phase shift is added to the Fourier phase spectrum, such that

$$s_{ap}(t) = e^{j\phi} s_a(t), \tag{4}$$

and,

$$S_{ap}(\omega) = e^{j\phi} S_a(\omega). \tag{5}$$

Then the magnitude spectrum of the windowed version of this signal, $s_{apw}(t)$, given by

$$|S_{apw}(\omega)| = |S_a(\omega) * W(\omega)|, \tag{6}$$

is independent of the phase rotation.

This phase invariance does not hold for real windowed signals. The real signal corresponding to $s_{ap}(t)$ can be written as

$$s_p(t) = (1/2)\{s_a(t)e^{j\phi} + s_a(t)e^{-j\phi}\}, \tag{7}$$

where $s_a^*(t)$ is the complex conjugate of $s_a(t)$. The spectrum of the windowed version of this signal is

$$S_{pw}(\omega) = (1/2)\{e^{j\phi} S_a(\omega)*W(\omega) + e^{-j\phi} S_a^*(-\omega)*W^*(\omega)\}. \tag{8}$$

The aliasing between positive and negative frequency components appears in the presence of the exponential term $e^{j\phi}$ and its conjugate. The magnitude spectrum of this signal is a function of the angle ϕ.

In the case of sums of periodic signals, as in Eq. 1, this phase invariance implies the magnitude spectrum of an analytic signal will be constant versus time, since this phase shift is equivalent to a time delay. For other signals, this phase shift can significantly change the appearance of the signal. An interesting example is shown in Fig. 1, where an ideal impulse function is shown for various values of the phase. In time series analysis, where continuous signals are always represented by as small a segment of the signal as possible, the analytic signal becomes a natural choice for improved resolution.

LINEAR PREDICTION USING ANALYTIC SIGNALS

The phase invariance property of analytic signals developed in the previous sections can be extended to the case of linear prediction. Let the phase rotated analytic sequence, $s_{ap}(nT)$, be defined as the sampled data sequence corresponding to Eq. 4. The linear prediction coefficients, $\{a_i\}$, for a complex time series can be computed from the Yule-Walker equation [1]:

$$R(0,r) = \sum_{i=1}^{p} a_i R^*(r,i), \quad 1 \le r \le p, \tag{9}$$

where R(r,i), the short-term autocorrelation function of a complex sequence s(nT), is defined as

$$R(r,i) = \sum_{m=0}^{N-1} s((m-r)T)s^*((m-i)T). \tag{10}$$

The short-term autocorrelation function of $s_{ap}(nT)$, computed from Eq. 10, is found to be

$$R_{ap}(r,i) = \sum_{m=0}^{N-1} e^{j\phi} s_a((m-r)T) \times e^{-j\phi} s_a^*((m-i)T) = R_a(r,i). \tag{11}$$

The short-term autocorrelation function for the analytic phase shifted signals is thus independent of the phase angle, ϕ. This result is not altogether unexpected, as linear prediction, by design, is blind to the phase of the signal. The LPC parameters determine an FIR filter whose inverse, when stable, is constrained to be of minimum phase [1]. It is easy to show [5] that the real signal equivalent of Eq. 4 is dependent on the phase angle, ϕ.

FREQUENCY ESTIMATES OF TWO SINEWAVES

While the analytic signal can provide an exact estimate of the frequency of a single sinewave [5], that is, independent of both phase and window length, this is just a special case. A more interesting case involves two sinewaves embedded in noise. Let the real signal to be estimated, s(nT), be defined as the sum of two cosinewaves plus additive noise,

$$s(nT) = \cos(\omega_1 nT + \phi_1) + \cos(\omega_2 nT + \phi_2) + Gv(nT). \tag{12}$$

The sequence v(nT) represents additive, zero mean, white Gaussian noise whose variance is unity. The signal-to-noise ratio is given by

$$SNR = 1/G^2 , \tag{13}$$

assuming independent sinewaves. Since this is actually a two-pole signal, a fourth order real estimator and a second order complex estimator will be used. The LPC polynomial will be computed as the frequency estimates taken as the values of the zero frequencies.

In Figs. 2 through 5, the variance of the estimators are compared for the first frequency. The data was obtained using the Burg algorithm. The frequencies of the sinewaves are 367 Hz and 859 Hz, while the phases are set to zero. The frame length is 20 points at an 8 KHz sampling rate for the real signal, and the signal-to-noise ratio is 40 dB. Because the analytic signal occupies half the bandwidth of the real signal, the complex signal is downsampled for LPC analysis. In this case, the window length for the complex signal (sampled at 4 kHz) is 10 samples.

In. Fig. 2, the variance of the frequency estimate for the first sinewave is plotted versus window length, while in Fig. 3, the variance is plotted versus signal-to-noise ratio. In Fig. 4, the variance is plotted versus π_1, while π_2 is zero. Finally, in Fig. 5, these experiments were repeated for the autocorrelation method. In this case, the sinewaves were more closely spaced, at frequencies of 414 Hz and 671 Hz. It should be emphasized that the reasons for the improvement in performance gained by using the analytic signal are, in general, independent of the particular LPC analysis algorithm.

Another reason for the superior performance of the analytic signal, as observed by Jackson, et. al. [3], is related to the Hilbert transform process. A sinewave travels through various zero crossings, which in the presence of additive noise, provide little information, since the "localized" signal-to-noise ratio at a zero crossing is very small. The analytic signal, which in this case is sum of complex exponentials, manages to stay above the noise level, allowing more data to make a consistent contribution to the LPC analysis process.

CONCLUSIONS

This paper has presented a new result concerning analytic signals. The analytic signal representation has been shown to be a natural choice for time series analysis. A property of the analytic time series, called phase invariance, allows analytic signal-based spectral estimators to achieve higher resolution than their real signal counterparts. This property implies that the magnitude spectrum of a periodic analytic signal computed from a frame-based spectral analysis will not vary as a function of time, or equivalently, as a function of the position of the window. Thus, the spectral estimates obtained from parametric spectral estimates are more consistent, especially in the presence of additive noise.

REFERENCES

1. S. M. Kay and S. L. Marple, Jr., "Spectrum Analysis - A Modern Perspective," Proc. of IEEE, vol. 69, no. 11, pp. 1380-1419, Nov. 1981.
2. W.T. Hartwell, "An Alternate Approach to the Use of Discrete Fourier Transforms," presented at the IEEE Workshop on FFT Processing, Harriman, New York, Oct. 1986.
3. L. Jackson, D. W. Tufts, F. W. Soong, and R. M. Rao, "Frequency Estimation by Linear Prediction," in Proc. 1978 IEEE Int. Conf. Acoust., Speech, Signal Proc., pp. 352-356, April 1978.
4. T.V. Ananthapadmanabha and B. Yegnanarayana, "Epoch Extraction from the Linear Prediction Residual for Identification of Closed Glottis Interval," IEEE Trans. Acoust., Speech, Signal Proc., vol. ASSP-27, no. 4, pp. 309-319, Aug. 1979.
5. J. Picone, Analytic Signal Processing, Ph. D. Thesis, Illinois Institute of Technology, Chicago, Illinois, 1983.
6. M. Schwartz and L. Shaw, Signal Processing, McGraw-Hill Book Co., New York, 1975.

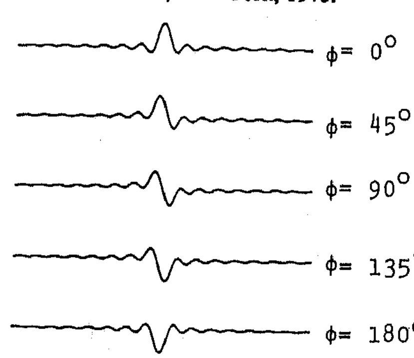

Fig. 1. Phase Rotation of an Ideal Impulse

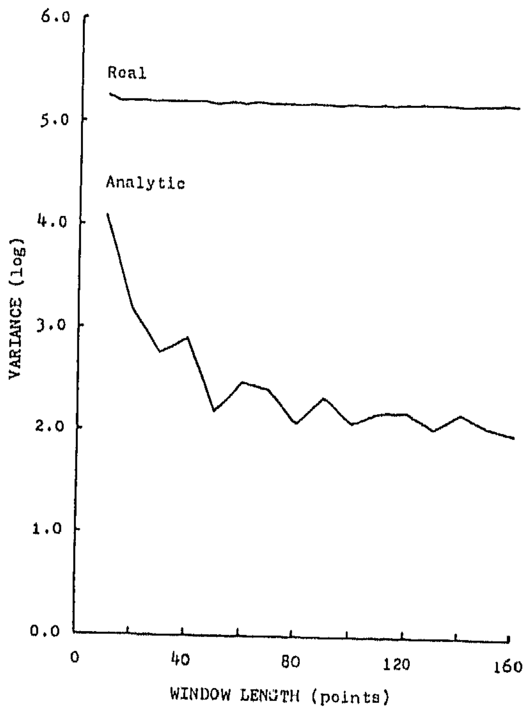

Fig. 2. Dependence of the Variance of the Frequency Estimates on Window Length (SNR = 20 dB)

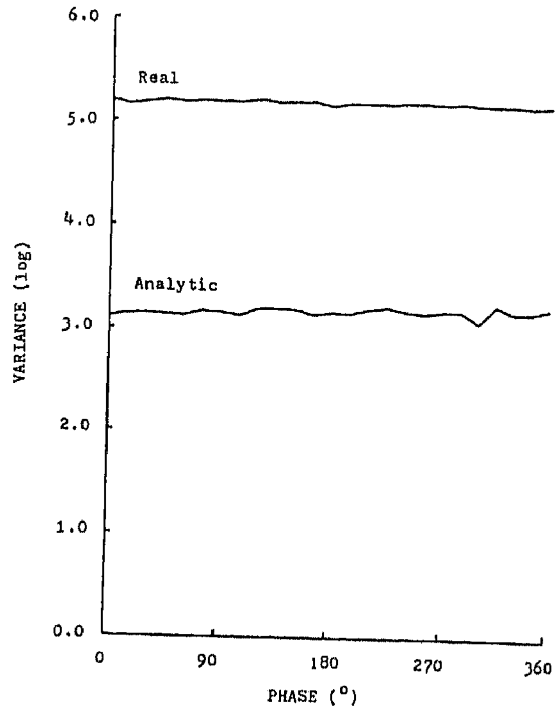

Fig. 4. Dependence of the Variance of the Frequency Estimates on Phase (SNR = 20 dB)

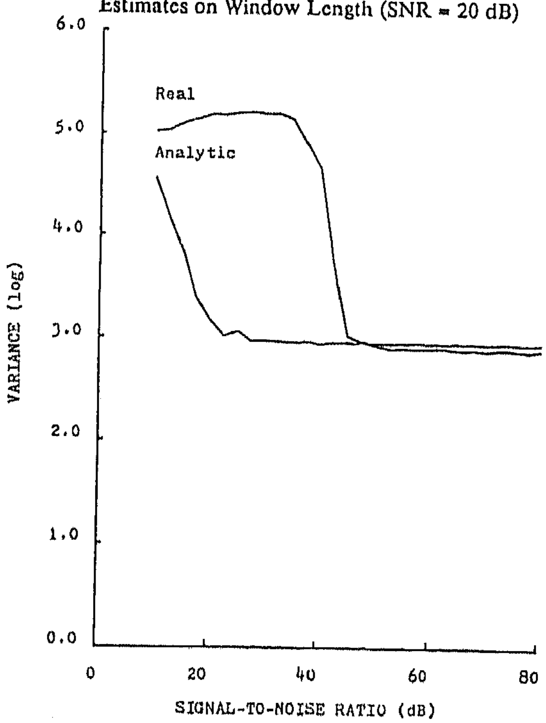

Fig. 3. Dependence of the Variance of the Frequency Estimates on Signal-to-Noise Ratio (N = 20 points)

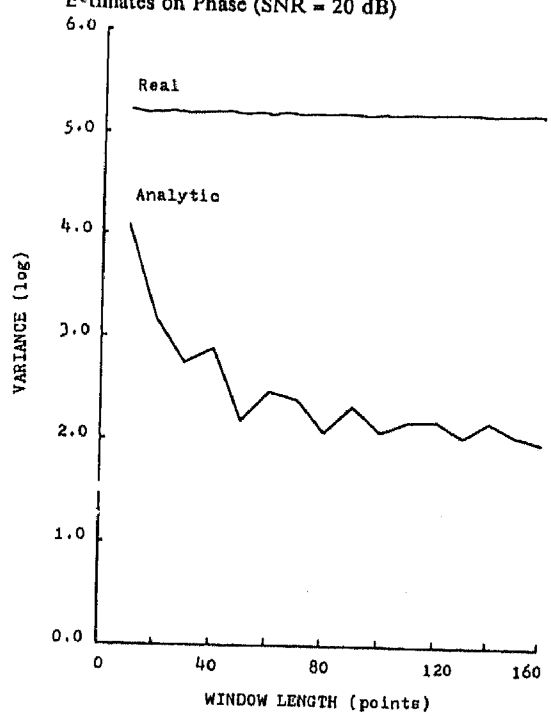

Fig. 5. Dependence of the Variance of the Frequency Estimates on Window Length for the Autocorrelation Method (SNR = 20 dB)

THE SENSITIVITY OF BEAMFORMING PROBLEMS

George Cybenko

Department of Computer Science
Tufts University
Medford, MA 02155

1. ABSTRACT

So-called eigenanalysis techniques hold great promise for solving high resolution beamforming and harmonic retrieval problems. At present however, it is not well understood how array geometries or specific problem instances affect the sensitivity of the frequency retrieval problem.

In this paper, we summarize results about the sensitivity of frequency retrieval as a function of the problem instance. It is seen that two factors are most important - the ratio of maximal to minimal sinusoid amplitudes and the value of the Vandermonde determinant based on the complex frequencies that make up the signal. Examples illustrate these findings.

2. INTRODUCTION

Eigenanalysis methods for the harmonic retrieval problem are extremely attractive in array processing and beamforming applications. In theory (assuming perfect and exact signal information), these techniques generate exact solutions. An important property of any numerical problem however is the solution's sensitivity to perturbations in the input data.

In this paper we summarize work showing that for a simple model problem, namely that of a signal composed of a sum of sinusoids with varying amplitudes and frequencies, we can determine the solution's sensitivity using a posteriori information. It is seen that the ratio of amplitudes between maximal and minimal energy harmonic components together with a certain determinant effectively characterize the sensitivity of a problem.

3. PROBLEM STATEMENT

Our basic problem formulation is taken from Pisarenko's fundamental work [1]. Suppose we observe a signal

$$s_j \quad \text{for } -\infty < j < \infty.$$

Assume it has the form

$$s_j = \sum_{k=1}^{p} \alpha_k \exp(\mathrm{i}\theta_k\, j) + \eta_j \tag{1}$$

where α_k are complex constants and θ_k are real and distinct. Here η_j is additive white noise with variance σ^2. Forming the asymptotic signal correlations

$$\rho_n = \lim_{N \to \infty} \frac{1}{2N} \sum_{k=-N}^{N} \bar{s}_j\, s_{j+n} \tag{2}$$

it can be shown that

$$\rho_n = \sum_{k=1}^{p} |\, \alpha_k |^2 \exp(\mathrm{i}\theta_k\, n) + \delta_{0k}\, \sigma^2. \tag{3}$$

Now defining the correlation matrix, T_p, according to

$$T_p = (t_{ij}) \quad \text{with } 1 \le i, j \le p+1 \tag{4}$$

with

$$t_{ij} = \rho_{i-j}$$

The fundamental question we address here is

> Suppose that T_p and $\tilde{T}_p$ correspond via the above constructions to two signals s and $\tilde{s}$, with the same p and σ^2. If T_p and $\tilde{T}_p$ are close to one another, say
>
> $$\| T_p - \tilde{T}_p \| = \epsilon$$
>
> how small should we expect
>
> $$|\exp(\mathrm{i}\theta_k) - \exp(\mathrm{i}\tilde{\theta}_k)|$$
>
> to be (assuming an optimal ordering of these quantities)?

4. SUMMARY OF RESULTS

It can be shown that two factors characterize the sensitivity of the reconstructed frequencies and amplitudes:

1. the ratio

$$\frac{\max_i |\alpha_i|}{\min_i |\alpha_j|}$$

of largest to smallest amplitudes;

2. the condition number of the Vandermonde matrix, V, where

$$v_{jk} = \exp(i\theta_j (k-1))$$

Recall that the condition number of a nonsingular matrix A is the product of norms

$$\kappa(A) = \| A \| \, \| A^{-1} \|$$

While the relative sizes between the signal amplitudes is predictably an important factor in solution sensitivity, the condition number of the Vandermonde matrix is perhaps surprising.

Some details of the derivation were presented in [2] and the reader is referred to that paper for a more complete treatment.

5. DISCUSSION

The two factors we have identified as playing an important role in the sensitivity of beamforming problems are amplitude ratios and frequency distribution. Note that in the case where the sinusoidal frequencies are uniformly distributed around the unit circle, the Vandermonde matrix is precisely the DFT matrix and is within a normalization constant, a unitary matrix. Thus in that case, the condition number is one and the problem is optimally conditioned. The Vandermonde matrix will have arbitrarily large condition number as the frequencies coalesce or cluster.

There are two interpretations or uses for these observations:

1. Any technique for solving beamforming problems can monitor the Vandermonde matrix of the frequencies and compute its condition number. If this number is large, the solution can be flagged as being possibly unreliable. If the number is small, the solution is probably highly accurate.
2. Even in the presence of exact information, the basic beamforming problem can give unreliable results because of finite observation records. Hence, there is a fundamental limitation to the accuracy of any technique for solving harmonic retrieval problems.

References

1. V.F. Pisarenko, "The retrieval of harmonics from a covariance function", *Geophysics J.R. Astro. Soc.*, Vol. 33, pp. 347-366, 1973.
2. G. Cybenko, "The conditioning of eigenvector methods for beamforming problems", *Proceedings of Princeton Conference on Information Systems and Sciences*, Princeton, NJ, 1986.