Name: ____

Problem	Points	Score
1	40	
2	40	
3	20	
Total	100	

Notes: The exam is closed book and closed notes.

(40 pts) Problem No. 1: Using least squares analysis, find the optimal value of z_1 in terms of the covariance values (e.g., $cov[1,2] = \sum_{i=0}^{N-1} y[n-1]y[n-2]$) for the following equation:

$$y[n] = z_0 + z_1 y[n-1]\ .$$

Assume $y[n]$ represents a vector of data:

$$y[n]\colon \left[y[0], y[1], y[2], \dots, y[N]\right]\ .$$

The signal $y[n-1]$ can be represented as a delayed version of $y[n]$:

$$y[n-1]\colon \left[y[-1], y[0], y[1], \dots, y[N-1]\right]\ .$$

Also, assume $z_0 = 0$.

(40 pts) Problem No. 2: Find the solution to the given differential equation:

$$\ddot{y} - 5\dot{y} + 6y = 0$$

Note that $\dot{y} = dy/dt$ and $\ddot{y} = d^2y/dt^2$. Assume $y(0) = 1$ and $\dot{y}(0) = 1$.

(20 pts) Problem No. 3: Theorem 10.1.4 states the Cauchy-Schwarz Inequality as: "If $\boldsymbol{v}$ and $\boldsymbol{w}$ are two vectors in an inner product space V, then: $\langle \boldsymbol{v}, \boldsymbol{w} \rangle^2 \leq \|\boldsymbol{v}\|^2 \|\boldsymbol{w}\|^2$. Moreover, equality occurs if and only if one of $\boldsymbol{v}$ and $\boldsymbol{w}$ is a scalar multiple of the other."

First, state the significance of this theorem in words. What does it mean? How does it relate to the dot product? What are the practical implications of this theorem?

Next, prove this theorem following this outline: Let $\|\boldsymbol{v}\| = a$ and $\|\boldsymbol{w}\| = b$. Expand $\|b\boldsymbol{v} - a\boldsymbol{w}\|^2$ and $\|b\boldsymbol{v} + a\boldsymbol{w}\|^2$ and note that these must be ≥ 0. Compare these two equations to establish a bound (range of values) for $\langle \boldsymbol{v}, \boldsymbol{w} \rangle$, and then show that the theorem must be true.