Name: _____

Problem	Points	Score
1	40	
2	30	
3	30	
Total	100	

Notes: The exam is closed book and closed notes.

(40 pts) Problem No. 1:

In various assignments we have explored curve fitting using the minimum number of data points (e.g., two equations and two unknowns). But how can we exploit more data? Assume you are given the input matrix, x, and output matrix, y, shown:

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

The best "linear" estimate of the coefficients of the model, $Y = Xa$, where $a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, can be found from the equation $(X^T X)a = X^T Y$. Find the best values for a. Show all your work.

(30 pts) Problem No. 2:

Given the matrices: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find the rank A, B, and the rank of the matrices the result from the following computations: (a) $A + B$, (b) $A - B$, (c) AB. Explain why your answers make sense and what are the implications of these results.

(30 pts) Problem No. 3:

Demonstrate whether these are linear transformations:

(a) $T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

(b) $T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} ax_1 & b \\ c & dx_2 \end{bmatrix}$,

(c) $T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ by_2 & 0 \end{bmatrix}$.

Note that $T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is shorthand notation for the system of equations:

$$y_1 = ax_1 + bx_2$$
$$y_2 = cx_1 + dx_2$$

where a, b, c, and d are constants. You must do this by demonstrating that $T\begin{bmatrix} qx_1 \\ qx_2 \end{bmatrix} = qT\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.