**ENGR 2013: Engineering Analysis and Applications**

**Laboratory No. 11: How Can We Characterize and Remove Noise in a Signal?**

**Goal:** The goal of this lab is to introduce you to some very basic calculations in statistics, and to understand how they model simple signals like Gaussian white noise and sinewaves. We are going to build on what you learned in lab no. 4 and lab no. 5.

**Preliminary Work:** Study how to generate Gaussian white noise using Python’s random.gauss function:

*https://www.geeksforgeeks.org/random-gauss-function-in-python/*

Generate Gaussian noise with a mean (average value) of zero and a variance of one.

Modify your code in lab no. 5 to produce this signal:

$$y\left(n\right)=x(n)+w(n)$$

where $x\left(n\right)=g\_{1}\sin(\left(2πf\_{1}(n/f\_{s})\right))$ and $w\left(n\right)=g\_{2}\*random.gauss()$.

Write a simple Python program that generates values of $y\left(n\right)$ for the range [0, 1000]. Set $f\_{1}=1.0 Hz $and $f\_{s}=10 Hz$. For $w\left(n\right)$, use the function random.gauss to generate zero-mean unit variance Gaussian white noise (as described above).

**Tasks:**

1. Set $g\_{1}=0$ and $g\_{2}=1$, and compute the correlation coefficient between $x(n)$ and $w(n)$, and between $y(n)$ and $w(n)$. Repeat this for $g\_{2}=1, 10, 100$. Did the correlation coefficient change?
2. Using the parameters in (1), compute the autocorrelation function for $K=50$ and $N=500$:

$R\left[k\right]=(\frac{1}{N})\sum\_{n=0}^{N-1}y\left(n\right)y(n+k),   for k=[0,K]$ ,

Explain what you observe and how this relates to (1).

1. Repeat (2) for $g\_{1}=[0.01, 0.1, 1, 10$] while keeping $g\_{2}=1$ . What do you observe? Why? How does the correlation coefficient relate to the autocorrelation function?
2. Set $g\_{1}=0$ and $g\_{2}=1$. Compute the mean, standard deviation, and variance of $y(n)$. Repeat for $g\_{2}=1, 2, 4, 8, 16$. Explain what you observe. How is the standard deviation related to $g\_{2}$?
3. Set $g\_{2}=0$ and $g\_{1}=1, 2, 4, 8, 16$. Explain whether these numbers match your theoretical predictions.
4. Set $g\_{2}=1$ and $g\_{1}=1.$ Compute the mean, standard deviation, and variance of $y(n)$. Explain from a theoretical point of view whether these numbers make sense (Hint: is the signal correlated with the noise? Does this make a difference?).
5. Add a second sinewave to $y(n)$: $g\_{3}\sin(\left(2πf\_{2}(n/f\_{s})\right))$. Set $f\_{2}=2.0 Hz$ and $g\_{3}=1$. Repeat (6). How are the variance calculations impacted? Why? What happens if you set $f\_{2}=1.77 Hz$?

The TAs will ask you to explain your results, and you will be judged on your ability to explain the reasons why you are observing the outputs you generated.

**Summary:**

Correlation operations are very useful in extracting signals from noise. In your Signals and Systems class, you will learn how to explain your observations in this lab using the concept of the frequency domain. You will learn about the relationship between the Fourier Transform and the autocorrelation function.