Name:

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| --- | --- | --- |
| Problem | Points | Score |
| 1 | 25 |  |
| 2a | 25 |  |
| 2b | 25 |  |
| 3 | 25 |  |
| Total | 100 |  |

Notes: The exam is closed book and closed notes.

**(25 pts) Problem No. 1**:

Show that: $det\left[\begin{matrix}0&I\_{n}\\I\_{m}&0\end{matrix}\right]=(-1)^{nm}$.

**Problem No. 2**:

**(25 pts)** (a) Find the eigenvalues for $A=\left[\begin{matrix}3&-2\\4&-1\end{matrix}\right]$. Explain why this answer makes sense.

**(25 pts)** (b) Find the eigenvalues and eigenvectors for $A=\left[\begin{matrix}2&1\\-1&4\end{matrix}\right]$. Explain if this answer makes sense.

**(25 pts) Problem No. 3**:

A square with sides if length 1 in a 2D space has an area of $1x1=1$. A (unit) cube with sides of length 1 in a 3D space has a volume of $1x1x1=1$. It can be argued that the analog of a unit cube in 4D space would be an object formed by vertices (1,0,0,0), (0,1,0,0), (0,0,1,0) and (0,0,0,1) and would enclose a ’volume’ of $1x1x1x1=1$ (or more generally $r^{n}$ where $r$ is the length of a side). We refer to this shape as a hypercube (a cube in $n$-dimensions).

How might you prove this using concepts developed in this section of the course? What happens if the sides are no longer perpendicular (a 4D parallelopiped)?