Name: ____

Problem	Points	Score
1(a)	20	
1(b)	10	
1(c)	10	
2	40	
3	20	
Total	100	

Notes:

(1) The exam is closed books and notes except for one double-sided sheet of notes.

(2) Please indicate clearly your answer to the problem.

(3) Note that ungrammatical sentences, incoherent statements, or general illegible scratches will get zero credit.

(4) If I can’t read or follow your solution, it is wrong, and no partial credit will be awarded.

Problem No. 1: Consider a two-class discrete distribution problem:

$$\omega_1: \{[0,0], [2,0], [2,2], [0,2]\}$$
$$\omega_2: \{[1,1], [2,1], [1,2], [3,3]\}$$

(20 pts) (a) Compute the minimum achievable error rate by a linear machine (hint: draw a picture of the data). Assume the classes are equiprobable.

(10 pts) (b) Assume the priors for each class are: $P(\omega_1) = \alpha$ and $P(\omega_2) = 1 - \alpha$. Sketch $P(E)$ as a function of α for a maximum likelihood classifier based on the assumption that each class is drawn from a multivariate Gaussian distribution. Compare and contrast your answer with your answer to (a). Be very specific in your sketch and label all critical points. Unlabeled plots will receive no partial credit.

(10 pts) (c) Assume you are not constrained to a linear machine. What is the minimum achievable error rate that can be achieved for this data? Is this value different than (a)? If so, why? How might you achieve such a solution? Compare and contrast this solution to (a).

Problem No. 2: Suppose we have a random sample $X_1, X_2, ..., X_n$ where:

- $X_i = 0$ if a randomly selected student does not own a laptop, and
- $X_i = 1$ if a randomly selected student does own a laptop.

(40 pts) Assuming that the X_i are independent Bernoulli random variables with unknown parameter p:

$$p(x; p) = (p)^{x_i}(1-p)^{1-x_i}$$

where $x_i = 0 \; or \; 1$ and $0 < p < 1.$ Find the maximum likelihood estimator of p, the proportion of students who own a laptop.

Problem No. 3: You are given a three minute section of an mp3 encoded music signal that has been corrupted by zero-mean Gaussian noise. Explain how you would apply the concepts learned in this course thus far to better estimate the spectrum of the signal. For example, you could simply take the entire signal (180 seconds) and compute the power spectrum using an FFT. Could you do better? How? Why? What concepts discussed in this course would be relevant to this problem?