

Name: _____

Problem	Points	Score
1(a)	20	
1(b)	20	
1(c)	10	
2(a)	20	
2(b)	30	
Total	100	

Notes:

- (1) The exam is closed books and notes except for one double-sided sheet of notes.
- (2) Please indicate clearly your answer to the problem.
- (3) Note that ungrammatical sentences, incoherent statements, or general illegible scratches will get zero credit.
- (4) If I can't read or follow your solution, it is wrong, and no partial credit will be awarded.

Problem No. 1: Consider two probability distributions representing a 2-class problem:

$$p(x|\omega_1) = \begin{cases} (\frac{2}{\alpha^2})x + (\frac{2}{\alpha}) & -\alpha \leq x < 0 \\ 0 & \text{elsewhere} \end{cases} \quad p(x|\omega_2) = \begin{cases} x + 1, & -1 \leq x < 0, \\ -x + 1, & 0 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

where $P(\omega_1) = 3/4$ and $P(\omega_2) = 1/4$.

- (20 pts) Sketch the probability of error as a function of α . Very carefully label your graph and be as precise as possible. Hint: sketch these distributions.
- (20 pts) For what value of α is the probability of error a maximum? Draw a sketch of the corresponding distributions to justify your choice.
- (10 pts) Suppose you weight your errors such that: $\lambda_{11} = \lambda_{22} = 0.1$ and $\lambda_{12} = \lambda_{21} = 1.0$. How would your answers to (a) and (b) change? Be as precise as possible in your arguments.

Problem No. 2: Suppose we have a discrete random variable, X , that takes on one of two values, 0 or 1, with the following probabilities:

$$p(x_i) = \begin{cases} 1 - \alpha & x = 0 \\ \alpha & x = 1 \end{cases}$$

- (20 pts) What is the mean of this distribution?
- (30 pts) What is the maximum likelihood estimate of α ? Justify your answer.