Name: ____

Problem	Points	Score
1(a)	20	
1(b)	20	
2(a)	20	
2(b)	20	
3	20	
Total	100	

Notes:

(1) The exam is closed books and notes except for one double-sided sheet of notes.

(2) Please indicate clearly your answer to the problem.

(3) If I can't read or follow your solution, it is wrong and no partial credit will be awarded.

Problem No. 1: Let $p(x \mid \omega_i) \sim N(\mu_i, \sigma^2)$ for a two-category one-dimensional problem with $P(\omega_1) = P(\omega_2) = 1/2$.

(a) Show that the minimum probability of error is given by: $P_e = \frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-u^2/2} du$ where $a = \left|\mu_2 - \mu_1\right| / 2\sigma$.

(b) Use the inequality $P_e = \frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-t^2/2} dt \le \frac{1}{\sqrt{2\pi}a} e^{-a^2/2}$ to show that P_e goes to zero as $\left|\mu_2 - \mu_1\right| / \sigma$ goes to infinity.

Problem No. 2: Given a two-class two-dimensional classification problem (x = $\{x_1,x_2\}$) with the following parameters (uniform distributions):

$$p(\mathrm{x} \mid \omega_1) = \begin{bmatrix} 1 & \begin{Bmatrix} -3/4 \le x_1 \le 1/4 \\ -3/4 \le x_2 \le 1/4 \end{Bmatrix} \\ 0 & elsewhere \end{bmatrix} \qquad p(\mathrm{x} \mid \omega_2) = \begin{bmatrix} 1 & \begin{Bmatrix} 0 \le x_1 \le 1 \\ 0 \le x_2 \le 1 \end{Bmatrix} \\ 0 & elsewhere \end{bmatrix}$$

where $P(\omega_1) = P(\omega_2) = 1/2$.

(a) Write the Bayes decision rule for this case (hint: draw the decision boundary). Is this solution unique? Explain.

(b) Compute the probability of error.

Problem No. 3: Let x have a uniform density: $p(x|\theta) = \begin{cases} 1/\theta & 0 \le x \le \theta \\ 0 & otherwise \end{cases}$. Suppose that n samples $D = \{x_1, x_2, \ldots, x_n\}$ are drawn independently from $p(x|\theta)$. Derive an expression for the maximum likelihood estimate of θ. Hint: compute the likelihood of the data given θ and differentiate. Discuss what happens to this estimate as $n \to \infty$.