Name:

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| --- | --- | --- |
| Problem | Points | Score |
| 1(a) | 30 |  |
| 1(b) | 10 |  |
| 2 | 30 |  |
| 3 | 30 |  |
| Total | 100 |  |

Notes:

1. The exam is closed books and notes except for one double-sided sheet of notes.
2. Please indicate clearly your answer to the problem.
3. If I can’t read or follow your solution, it is wrong and no partial credit will be awarded.

**Problem No. 1**: Consider two probability distributions defined by:

 and .

(a) Sketch the probability of error, P(E), for a maximum likelihood classifier as a function of  and . Label all critical points.

(b) Suppose you estimated these distributions to be Gaussians rather than uniform by analyzing a large amount of training data drawn from each distribution. How would your result in (a) change?

**Problem No. 2**: Let x have a uniform density: . Suppose that n samples  are drawn independently from . Derive an expression for the maximum likelihood estimate of *θ*. Hint: compute the likelihood of the data given *θ* and differentiate. Discuss what happens to this estimate as .

**Problem No. 3**: A zero-mean unit variance discrete-time Gaussian white noise signal, x[n], is applied to a digital filter: . Assume you only have access to the output of this filter, but you do know the form of the filter (you just don’t know the specific value of *α*), and you can assume the input is zero-mean Gaussian white noise. Derive or explain how you would construct a maximum likelihood estimate of the filter coefficient. Hint: think about the pdf for the difference of two random variables. Second hint: think about the role correlation can play in this estimate.