Name:

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| Problem | Points | Score |
| 1(a) | 35 |  |
| 1(b) | 10 |  |
| 2(a) | 35 |  |
| 2(b) | 10 |  |
| 3 | 10 |  |
| Total | 100 |  |

Notes:

1. The exam is closed books and notes except for one double-sided sheet of notes.
2. Please indicate clearly your answer to the problem.
3. Note that ungrammatical sentences, incoherent statements, or general illegible scratches will get zero credit.
4. If I can’t read or follow your solution, it is wrong, and no partial credit will be awarded.

**Problem No. 1**: Suppose a sample $x\_{1},x\_{2},…,x\_{n}$ is modelled by a Poisson distribution:
 $p\left(x;λ\right)=\frac{λ^{x}}{x!}e^{-λ}$.

(35 pts) (a) Find the maximum likelihood estimate of $λ$.

(10 pts) (b) Suppose you have a two-class problem where $p\left(ω\_{1}\left|x\right.\right) $and $p\left(ω\_{2}\left|x\right.\right) $are modelled as Poisson distributions, with means $λ\_{1}$ and $λ\_{2}$. Explain how you would design a maximum likelihood classifier assuming uniform priors that minimizes the error rate. Set up the corresponding integrals to calculate the probability of error but don’t evaluate them. Describe the expected result and discuss ways you might decrease the probability of error.

**Problem No. 2**: Consider two probability distributions defined by:

$p\left(x|ω\_{1}\right)=\left\{\begin{matrix}1,&α-{1}/{2}\leq x\leq α+1/2, \\0&elsewhere, \end{matrix}\right\}$

$$p\left(x|ω\_{2}\right)=\left\{\begin{matrix}1,&-{1}/{2}\leq x\leq 1/2,\\0,&elsewhere.\end{matrix}\right\}$$

(35 pts) (a) Sketch the probability of error, $P\left(E\right)$, for a maximum likelihood classifier as a function of $α$. Assume equal priors.

(10 pts) (b) Sketch the probability of error, $P\left(E\right)$, for a maximum likelihood classifier as a function of $P\left(ω\_{1}\right)$.

**Problem No. 3**: A zero-mean unit variance discrete-time Gaussian white noise signal, $x\left(n\right)$, is applied to a digital filter: $H\left(z\right)={1}/{(1-αz^{-1})}$. Assume you only have access to the output of this filter, but you do know the form of the filter (you just don’t know the specific value of *α*), and you can assume the input is zero-mean Gaussian white noise.

(10 pts) (a) Derive or explain how you would construct a maximum likelihood estimate of the filter coefficient. Hint: think about the pdf for the difference of two random variables. Second hint: think about the role correlation can play in this estimate.