Name: ____

Problem	Points	Score
1(a)	20	
1(b)	20	
2(a)	20	
2(b)	20	
3(a)	20	
Total	100	

Notes:

(1) The exam is open books and notes, but no AI tools or Google searching please ☺ You can browse the class web site.

(2) Please indicate clearly your answer to the problem.

(3) Note that ungrammatical sentences, incoherent statements, or general illegible scratches will get zero credit.

(4) If I can't read or follow your solution, it is wrong, and no partial credit will be awarded.

Problem No. 1: Let's assume you have a 1D Gaussian source which generates random scalars (vectors of the form $[x_1]$). You observe the following data: [1], [2], [3].

(20 pts) (a) Using Bayesian estimation techniques, what is your best estimate of the mean based on these observations? Explain your assumptions clearly. Think carefully about all the unknowns in the problem.

(20 pts) (b) Now, suppose you observe a 4th value: [5]. How does this impact your estimate of the mean? Explain, being as specific as possible. Support your explanation with calculations, equations, and most importantly, theoretical justifications.

Problem No. 2: Consider two probability distributions defined by:

$$p(\omega_1|x_1,x_2) = \begin{Bmatrix} 1 & 0 \le x_1 \le 1, 0 \le x_2 \le 1 \\ 0 & elsewhere \end{Bmatrix}$$

$$p(\omega_2|x_1,x_2) = \begin{Bmatrix} 1 & \alpha \le x_1 \le 1+\alpha, \alpha \le x_2 \le 1+\alpha \\ 0 & elsewhere \end{Bmatrix}$$

(20 pts) (a) Assuming the prior probabilities, $P(\omega_1) = P(\omega_2) = 0.5$, sketch the probability of error, $P(E)$, for a maximum likelihood classifier as a function of α. Label all critical points.

(20 pts) (b) How does the shape of this plot change if $P(\omega_1) = 0.75$ and $P(\omega_2) = 0.25$? Sketch the new shape and label all critical points. Justify your answer.

Problem No. 3: Suppose we have a discrete random variable, X, that takes on one of two values, 0 or 1, with the following probabilities:

$$p(x_i) = \begin{Bmatrix} 1-\alpha & x = 0 \\ \alpha & x = 1 \end{Bmatrix}$$

(20 pts) (a) What is the maximum likelihood estimate of α? Justify your answer.