Name: ____

Problem	Points	Score
1(a)	30	
1(b)	10	
1(c)	10	
2(a)	20	
2(b)	10	
3	20	
Total	100	

Notes:

(1) The exam is closed books and notes except for one double-sided sheet of notes.

(2) Please indicate clearly your answer to the problem.

(3) Note that ungrammatical sentences, incoherent statements, or general illegible scratches will get zero credit.

(4) If I can't read or follow your solution, it is wrong, and no partial credit will be awarded.

Problem No. 1: Consider two probability distributions defined by:

$$p(\omega_1|x_1,x_2) = \begin{Bmatrix} 1/2, & 0 \le x_1 \le 1, 0 \le x_2 \le 1, \\ 1/2 & 2 \le x_1 \le 3, 2 \le x_2 \le 3, \\ 0 & elsewhere \end{Bmatrix}$$

$$p(\omega_2|x_1,x_2) = \begin{Bmatrix} 1/4, & \alpha \le x_1 \le 2+\alpha, \alpha \le x_2 \le 2+\alpha, \\ 0, & elsewhere \end{Bmatrix}$$

(30 pts) (a) Assuming the prior probabilities, $P(\omega_1) = P(\omega_2) = 0.5$, sketch the probability of error, P(E), for a maximum likelihood classifier as a function of α for $0 \le \alpha \le 1$. Clearly label all critical points for both the horizontal axis (independent variable) and vertical axis (dependent variable. You must compute accurate numbers for these – unlabeled plots will receive no credit.

(10 pts) (b) How does the shape of this plot change if $P(\omega_1) = 0.75$ and $P(\omega_2) = 0.25$? Sketch the new shape and label all critical points. Justify your answer.

(10 pts) (c) Compute the covariance matrix for (x_1, x_2). How does the covariance influence the minimum achievable error rate?

Problem No. 2: A coin is flipped 100 times. Given that there were 55 heads, find the maximum likelihood estimate for the probability p of heads on a single toss. Note that the probability distribution obeys a Bernoulli random variable with unknown parameter *p*:

$$p(55\ heads|p) = (p)^{55}(1-p)^{1-55}$$

where $0 < p < 1$.

(20 pts) (a) Find the maximum likelihood estimate of *p*.

(10 pts) (b) Describe how you would apply Bayesian parameter estimation to the problem of estimating p. How different would the results be?

Problem No. 3: You are given a signal plus zero-mean colored Gaussian additive noise (the noise does not have a flat power spectrum) – for example a recording of a concert in an outdoor arena. Your goal is to remove the noise or mitigate its effects.

(20 pts) Explain how you would apply the concepts learned in this course thus far to estimate the spectrum of the signal and the spectrum of the noise. If you had reliable estimates of these, how might you go about separating the two?