Name: _____

Problem	Points	Score
1(a)	30	
1(b)	10	
2(a)	30	
2(b)	10	
3	20	
Total	100	

Notes:

(1) The exam is closed books and notes except for one double-sided sheet of notes.

(2) Please indicate clearly your answer to the problem.

(3) If I can't read or follow your solution, it is wrong and no partial credit will be awarded.

Problem No. 1: Consider two probability distributions defined by:

$$p(\omega_1|x_1,x_2) = \begin{Bmatrix} 1 & 0 \le x_1 \le 1, 0 \le x_2 \le 1 \\ 0 & elsewhere \end{Bmatrix}$$

$$p(\omega_2|x_1,x_2) = \begin{Bmatrix} 1 & \alpha \le x_1 \le 1+\alpha, \alpha \le x_2 \le 1+\alpha \\ 0 & elsewhere \end{Bmatrix}$$

(30 pts) (a) Assuming the prior probabilities, $P(\omega_1) = P(\omega_2) = 0.5$, sketch the probability of error, P(E), for a maximum likelihood classifier as a function of α. Label all critical points.

(10 pts) (b) How does the shape of this plot change if $P(\omega_1) = 0.75$ and $P(\omega_2) = 0.25$? Sketch the new shape and label all critical points. Justify your answer.

Problem No. 2: Suppose we have a discrete random variable, X, that takes on one of two values, 0 or 1, with the following probabilities:

$$p(x_i) = \begin{Bmatrix} 1-\alpha & x=0 \\ \alpha & x=1 \end{Bmatrix}$$

(30 pts) (a) What is the maximum likelihood estimate of α? Justify your answer.

(10 pts) (b) Describe how this would change if you constructed a Bayesian estimate of α? (Hint: set up the equations that must be solved and discuss their implications.)

Problem No. 3: (20 pts) You are given a three minute section of an mp3 encoded music signal that has been corrupted by zero-mean Gaussian noise. Explain how you would apply the concepts learned in this course thus far to better estimate the spectrum of the signal. For example, you could simply take the entire signal (180 seconds) and compute the power spectrum using an FFT. Could you do better? How? Why? What concepts discussed in this course would be relevant to this problem?