Denoising Autoencoders (dA)

Note

This section assumes the reader has already read through <u>Classifying MNIST digits using Logistic Regression</u> and <u>Multilayer Perceptron</u>. Additionally it uses the following Theano functions and concepts: <u>T.tanh</u>, <u>shared variables</u>, <u>basic arithmetic ops</u>, <u>T.grad</u>, <u>Random numbers</u>, <u>floatX</u>. If you intend to run the code on GPU also read <u>GPU</u>.

Note

The code for this section is available for download <u>here</u>.

The Denoising Autoencoder (dA) is an extension of a classical autoencoder and it was introduced as a building block for deep networks in [Vincent08]. We will start the tutorial with a short discussion on Autoencoders.

Autoencoders

See section 4.6 of [Bengio09] for an overview of auto-encoders. An autoencoder takes an input $\mathbf{x} \in [0,1]^d$ and first maps it (with an *encoder*) to a hidden representation $\mathbf{y} \in [0,1]^{d'}$ through a deterministic mapping, e.g.:

$$y = s(Wx + b)$$

Where s is a non-linearity such as the sigmoid. The latent representation y, or **code** is then mapped back (with a *decoder*) into a **reconstruction** z of the same shape as x. The mapping happens through a similar transformation, e.g.:

$$\mathbf{z} = s(\mathbf{W}'\mathbf{y} + \mathbf{b}')$$

(Here, the prime symbol does not indicate matrix transposition.) \mathbf{z} should be seen as a prediction of \mathbf{x} , given the code \mathbf{y} . Optionally, the weight matrix \mathbf{W}' of the reverse mapping may be constrained to be the transpose of the forward mapping: $\mathbf{W}' = \mathbf{W}^T$. This is referred to as *tied weights*. The parameters of this model (namely \mathbf{W} , \mathbf{b} , \mathbf{b}' and, if one doesn't use tied weights, also \mathbf{W}') are optimized such that the average reconstruction error is minimized.

The reconstruction error can be measured in many ways, depending on the appropriate distributional assumptions on the input given the code. The traditional *squared error* $L(\mathbf{x}\mathbf{z}) = ||\mathbf{x} - \mathbf{z}||^2$, can be used. If the input is interpreted as either bit vectors or vectors of bit probabilities, *cross-entropy* of the reconstruction can be used:

$$L_H(\mathbf{x}, \mathbf{z}) = -\sum_{k=1}^{d} [\mathbf{x}_k \log \mathbf{z}_k + (1 - \mathbf{x}_k) \log(1 - \mathbf{z}_k)]$$

The hope is that the code \mathbf{y} is a *distributed* representation that captures the coordinates along the main factors of variation in the data. This is similar to the way the projection on principal components would capture the main factors of variation in the data. Indeed, if there is one linear hidden layer (the *code*) and the mean squared error criterion is used to train the network, then the k hidden units learn to project the input in the span of the first k principal components of the data. If the hidden layer is nonlinear, the auto-encoder behaves differently from PCA, with the ability to capture multi-modal aspects of the input distribution. The departure from PCA becomes even more important when we consider *stacking multiple encoders* (and their corresponding decoders) when building a deep auto-encoder [Hinton06].

Because y is viewed as a lossy compression of x, it cannot be a good (small-loss) compression for all x. Optimization makes it a good compression for training examples, and hopefully for other inputs as well, but not for arbitrary inputs. That is the sense in which an auto-encoder generalizes: it gives low reconstruction error on test examples from the same distribution as the training examples, but generally high reconstruction error on samples randomly chosen from the input space.

We want to implement an auto-encoder using Theano, in the form of a class, that could be afterwards used in constructing a stacked autoencoder. The first step is to create shared variables for the parameters of the autoencoder \mathbf{W} , \mathbf{b} and \mathbf{b}' . (Since we are using tied weights in this tutorial, \mathbf{W}^T will be used for \mathbf{W}'):

```
def __init__(
    self,
    numpy_rng,
    theano_rng=None,
   input=None,
    n_visible=784,
    n_hidden=500,
   W=None,
    bhid=None,
    bvis=None
):
    Initialize the dA class by specifying the number of visible units (the
   dimension d of the input ), the number of hidden units ( the dimension
    d' of the Latent or hidden space ) and the corruption Level. The
    constructor also receives symbolic variables for the input, weights and
    bias. Such a symbolic variables are useful when, for example the input
    is the result of some computations, or when weights are shared between
    the dA and an MLP layer. When dealing with SdAs this always happens,
    the dA on layer 2 gets as input the output of the dA on layer 1,
    and the weights of the dA are used in the second stage of training
    to construct an MLP.
    :type numpy_rng: numpy.random.RandomState
    :param numpy_rng: number random generator used to generate weights
    :type theano_rng: theano.tensor.shared_randomstreams.RandomStreams
    :param theano_rng: Theano random generator; if None is given one is
                 generated based on a seed drawn from `rng`
    :type input: theano.tensor.TensorType
    :param input: a symbolic description of the input or None for
                  standalone dA
    :type n_visible: int
    :param n_visible: number of visible units
    :type n hidden: int
    :param n_hidden: number of hidden units
    :type W: theano.tensor.TensorType
    :param W: Theano variable pointing to a set of weights that should be
              shared belong the dA and another architecture; if dA should
              be standalone set this to None
    :type bhid: theano.tensor.TensorType
    :param bhid: Theano variable pointing to a set of biases values (for
                 hidden units) that should be shared belong dA and another
                 architecture; if dA should be standalone set this to None
    :type bvis: theano.tensor.TensorType
    :param bvis: Theano variable pointing to a set of biases values (for
                 visible units) that should be shared belong dA and another
                 architecture; if dA should be standalone set this to None
    self.n_visible = n_visible
    self.n_hidden = n_hidden
    # create a Theano random generator that gives symbolic random values
    if not theano_rng:
        theano_rng = RandomStreams(numpy_rng.randint(2 ** 30))
    # note : W' was written as `W_prime` and b' as `b_prime`
    if not W:
        # W is initialized with `initial_W` which is uniformely sampled
        # from -4*sqrt(6./(n_visible+n_hidden)) and
        # 4*sqrt(6./(n_hidden+n_visible))the output of uniform if
        # converted using asarray to dtype
        # theano.config.floatX so that the code is runable on GPU
        initial W = numpy.asarray(
```

```
numpy_rng.uniform(
            low=-4 * numpy.sqrt(6. / (n_hidden + n_visible)),
            high=4 * numpy.sqrt(6. / (n_hidden + n_visible)),
            size=(n_visible, n_hidden)
        \verb|dtype=theano.config.floatX|\\
    W = theano.shared(value=initial_W, name='W', borrow=True)
if not bvis:
    bvis = theano.shared(
        value=numpy.zeros(
            n_visible,
            dtype=theano.config.floatX
        borrow=True
    )
if not bhid:
    bhid = theano.shared(
        value=numpy.zeros(
            n hidden.
            dtype=theano.config.floatX
        name='b'
        borrow=True
    )
self.W = W
# b corresponds to the bias of the hidden
self.b = bhid
# b_prime corresponds to the bias of the visible
self.b_prime = bvis
# tied weights, therefore W_prime is W transpose
self.W_prime = self.W.T
self.theano_rng = theano_rng
# if no input is given, generate a variable representing the input
if input is None:
    # we use a matrix because we expect a minibatch of several
    # examples, each example being a row
    self.x = T.dmatrix(name='input')
else:
    self.x = input
self.params = [self.W, self.b, self.b_prime]
```

Note that we pass the symbolic <u>input</u> to the autoencoder as a parameter. This is so that we can concatenate layers of autoencoders to form a deep network: the symbolic output (the y above) of layer k will be the symbolic input of layer k+1.

Now we can express the computation of the latent representation and of the reconstructed signal:

```
def get_hidden_values(self, input):
    """ Computes the values of the hidden layer """
    return T.nnet.sigmoid(T.dot(input, self.W) + self.b)

def get_reconstructed_input(self, hidden):
    """Computes the reconstructed input given the values of the hidden layer
    """
    return T.nnet.sigmoid(T.dot(hidden, self.W_prime) + self.b_prime)
```

And using these functions we can compute the cost and the updates of one stochastic gradient descent step:

```
def get_cost_updates(self, corruption_level, learning_rate):
    """ This function computes the cost and the updates for one training
    step of the dA """

    tilde_x = self.get_corrupted_input(self.x, corruption_level)
    y = self.get_hidden_values(tilde_x)
    z = self.get_reconstructed_input(y)
    # note : we sum over the size of a datapoint; if we are using
```

```
minibatches, L will be a vector, with one entry per
         example in minibatch
L = -T.sum(self.x * T.log(z) + (1 - self.x) * T.log(1 - z), axis=1)
# note : L is now a vector, where each element is the
         cross-entropy cost of the reconstruction of the
         corresponding example of the minibatch. We need to
         compute the average of all these to get the cost of
#
         the minibatch
cost = T.mean(L)
# compute the gradients of the cost of the `dA` with respect
# to its parameters
gparams = T.grad(cost, self.params)
# generate the list of updates
updates = [
    (param, param - learning_rate * gparam)
    for param, gparam in zip(self.params, gparams)
return (cost, updates)
```

We can now define a function that applied iteratively will update the parameters \underline{w} , \underline{b} and \underline{b} _prime such that the reconstruction cost is approximately minimized.

```
da = dA(
    numpy_rng=rng,
    theano_rng=theano_rng,
    input=x,
    n_visible=28 * 28,
    n_hidden=500
)
cost, updates = da.get_cost_updates(
    corruption_level=0.,
    learning_rate=learning_rate
train_da = theano.function(
   [index],
    cost,
    updates=updates,
    givens={
        x: train set x[index * batch size: (index + 1) * batch size]
)
start_time = timeit.default_timer()
############
# TRAINING #
############
# go through training epochs
for epoch in xrange(training_epochs):
    # go through trainng set
    c = []
    for batch_index in xrange(n_train_batches):
        c.append(train_da(batch_index))
    print 'Training epoch %d, cost ' % epoch, numpy.mean(c)
end_time = timeit.default_timer()
training_time = (end_time - start_time)
print >> sys.stderr, ('The no corruption code for file ' +
                      os.path.split(__file__)[1] +
' ran for %.2fm' % ((training_time) / 60.))
image = Image.fromarray(
    tile_raster_images(X=da.W.get_value(borrow=True).T,
                       img_shape=(28, 28), tile_shape=(10, 10),
                       tile_spacing=(1, 1)))
image.save('filters_corruption_0.png')
# start-snippet-3
# BUILDING THE MODEL CORRUPTION 30% #
```

```
rng = numpy.random.RandomState(123)
    theano_rng = RandomStreams(rng.randint(2 ** 30))
    da = dA(
        numpy_rng=rng,
        theano_rng=theano_rng,
        input=x,
        n_visible=28 * 28,
        n_hidden=500
    cost, updates = da.get_cost_updates(
        corruption_level=0.3,
        learning_rate=learning_rate
    train_da = theano.function(
        [index],
        cost,
        updates=updates,
        givens={
            x: train_set_x[index * batch_size: (index + 1) * batch_size]
    )
    start_time = timeit.default_timer()
    ############
    # TRAINING #
    ############
    # go through training epochs
    for epoch in xrange(training_epochs):
        # go through trainng set
        c = []
        for batch_index in xrange(n_train_batches):
            c.append(train da(batch index))
        print 'Training epoch %d, cost ' % epoch, numpy.mean(c)
    end_time = timeit.default_timer()
    training time = (end time - start time)
    print >> sys.stderr, ('The 30% corruption code for file ' +
                          os.path.split(__file__)[1] +
                           ran for %.2fm' % (training time / 60.))
    # end-snippet-3
    # start-snippet-4
    image = Image.fromarray(tile_raster_images(
        X=da.W.get_value(borrow=True).T;
        img_shape=(28, 28), tile_shape=(10, 10),
        tile_spacing=(1, 1)))
    image.save('filters_corruption_30.png')
    # end-snippet-4
    os.chdir('../')
if __name__ == '__main__':
    test_dA()
```

If there is no constraint besides minimizing the reconstruction error, one might expect an auto-encoder with \boldsymbol{n} inputs and an encoding of dimension \boldsymbol{n} (or greater) to learn the identity function, merely mapping an input to its copy. Such an autoencoder would not differentiate test examples (from the training distribution) from other input configurations.

Surprisingly, experiments reported in [Bengio07] suggest that, in practice, when trained with stochastic gradient descent, non-linear auto-encoders with more hidden units than inputs (called overcomplete) yield useful representations. (Here, "useful" means that a network taking the encoding as input has low classification error.)

A simple explanation is that stochastic gradient descent with early stopping is similar to an L2

regularization of the parameters. To achieve perfect reconstruction of continuous inputs, a one-hidden layer auto-encoder with non-linear hidden units (exactly like in the above code) needs very small weights in the first (encoding) layer, to bring the non-linearity of the hidden units into their linear regime, and very large weights in the second (decoding) layer. With binary inputs, very large weights are also needed to completely minimize the reconstruction error. Since the implicit or explicit regularization makes it difficult to reach large-weight solutions, the optimization algorithm finds encodings which only work well for examples similar to those in the training set, which is what we want. It means that the representation is exploiting statistical regularities present in the training set, rather than merely learning to replicate the input.

There are other ways by which an auto-encoder with more hidden units than inputs could be prevented from learning the identity function, capturing something useful about the input in its hidden representation. One is the addition of *sparsity* (forcing many of the hidden units to be zero or near-zero). Sparsity has been exploited very successfully by many [Ranzato07] [Lee08]. Another is to add randomness in the transformation from input to reconstruction. This technique is used in Restricted Boltzmann Machines (discussed later in Restricted Boltzmann Machines (RBM)), as well as in Denoising Auto-Encoders, discussed below.

Denoising Autoencoders

The idea behind denoising autoencoders is simple. In order to force the hidden layer to discover more robust features and prevent it from simply learning the identity, we train the autoencoder to *reconstruct* the input from a corrupted version of it.

The denoising auto-encoder is a stochastic version of the auto-encoder. Intuitively, a denoising auto-encoder does two things: try to encode the input (preserve the information about the input), and try to undo the effect of a corruption process stochastically applied to the input of the auto-encoder. The latter can only be done by capturing the statistical dependencies between the inputs. The denoising auto-encoder can be understood from different perspectives (the manifold learning perspective, stochastic operator perspective, bottom-up - information theoretic perspective, top-down - generative model perspective), all of which are explained in [Vincent08]. See also section 7.2 of [Bengio09] for an overview of auto-encoders.

In [Vincent08], the stochastic corruption process randomly sets some of the inputs (as many as half of them) to zero. Hence the denoising auto-encoder is trying to *predict the corrupted (i.e. missing) values from the uncorrupted (i.e., non-missing) values*, for randomly selected subsets of missing patterns. Note how being able to predict any subset of variables from the rest is a sufficient condition for completely capturing the joint distribution between a set of variables (this is how Gibbs sampling works).

To convert the autoencoder class into a denoising autoencoder class, all we need to do is to add a stochastic corruption step operating on the input. The input can be corrupted in many ways, but in this tutorial we will stick to the original corruption mechanism of randomly masking entries of the input by making them zero. The code below does just that:

```
def get_corrupted_input(self, input, corruption_level):
    """This function keeps ``1-corruption_level`` entries of the inputs the
    same and zero-out randomly selected subset of size ``coruption level`
    Note: first argument of theano.rng.binomial is the shape(size) of
            random numbers that it should produce
            second argument is the number of trials
            third argument is the probability of success of any trial
              ropability of 1 - ``corruption_level`` and 0 with `corruption_level``
             this will produce an array of 0s and 1s where 1 has a
             probability of 1 - `
             The binomial function return int64 data type by
             default. int64 multiplicated by the input
             type(floatX) always return float64. To keep all data
             in floatX when floatX is float32, we set the dtype of
             the binomial to floatX. As in our case the value of
             the binomial is always 0 or 1, this don't change the
             result. This is needed to allow the gpu to work
```

```
correctly as it only support float32 for now.
return self.theano_rng.binomial(size=input.shape, n=1,
                                p=1 - corruption level,
                                dtype=theano.config.floatX) * input
```

In the stacked autoencoder class (Stacked Autoencoders) the weights of the dA class have to be shared with those of a corresponding sigmoid layer. For this reason, the constructor of the dA also gets Theano variables pointing to the shared parameters. If those parameters are left to None, new ones will be constructed.

The final denoising autoencoder class becomes:

```
class dA(object):
    """Denoising Auto-Encoder class (dA)
    A denoising autoencoders tries to reconstruct the input from a corrupted
    version of it by projecting it first in a latent space and reprojecting
    it afterwards back in the input space. Please refer to Vincent et al., 2008
    for more details. If x is the input then equation (1) computes a partially
    destroyed version of x by means of a stochastic mapping q_D. Equation (2)
    computes the projection of the input into the latent space. Equation (3)
    computes the reconstruction of the input, while equation (4) computes the
    reconstruction error.
    .. math::
        \forall tilde\{x\} \sim q_D(\forall tilde\{x\} | x)
                                                                            (1)
       y = s(W \setminus tilde\{x\} + b)
                                                                            (2)
        x = s(W' y + b')
                                                                            (3)
        L(x,z) = -sum_{k=1}^d [x_k \setminus log z_k + (1-x_k) \setminus log(1-z_k)]
                                                                            (4)
    def __init__(
        self,
        numpy_rng,
        theano_rng=None,
        input=None,
        n_visible=784,
        n_hidden=500,
        W=None,
        bhid=None.
        bvis=None
    ):
        Initialize the dA class by specifying the number of visible units (the
        dimension d of the input ), the number of hidden units ( the dimension
        d' of the latent or hidden space ) and the corruption level. The
        constructor also receives symbolic variables for the input, weights and
        bias. Such a symbolic variables are useful when, for example the input
        is the result of some computations, or when weights are shared between
        the dA and an MLP layer. When dealing with SdAs this always happens,
        the dA on layer 2 gets as input the output of the dA on layer 1,
        and the weights of the dA are used in the second stage of training
        to construct an MLP.
        :type numpy_rng: numpy.random.RandomState
        :param numpy_rng: number random generator used to generate weights
        :type theano_rng: theano.tensor.shared_randomstreams.RandomStreams
        :param theano_rng: Theano random generator; if None is given one is
                     generated based on a seed drawn from `rng`
        :type input: theano.tensor.TensorType
        :param input: a symbolic description of the input or None for
                      standalone dA
        :type n_visible: int
        :param n visible: number of visible units
        :type n_hidden: int
```

```
:param n_hidden: number of hidden units
:type W: theano.tensor.TensorType
:param W: Theano variable pointing to a set of weights that should be
          shared belong the dA and another architecture; if dA should
          be standalone set this to None
:type bhid: theano.tensor.TensorType
:param bhid: Theano variable pointing to a set of biases values (for
             hidden units) that should be shared belong dA and another
             architecture; if dA should be standalone set this to None
:type bvis: theano.tensor.TensorType
:param bvis: Theano variable pointing to a set of biases values (for
             visible units) that should be shared belong dA and another
             architecture; if dA should be standalone set this to None
self.n_visible = n_visible
self.n hidden = n hidden
# create a Theano random generator that gives symbolic random values
if not theano_rng:
    theano_rng = RandomStreams(numpy_rng.randint(2 ** 30))
# note : W' was written as `W prime` and b' as `b prime`
if not W:
    # W is initialized with `initial_W` which is uniformely sampled
    # from -4*sqrt(6./(n_visible+n_hidden)) and
    # 4*sqrt(6./(n_hidden+n_visible))the output of uniform if
    # converted using asarray to dtype
    # theano.config.floatX so that the code is runable on GPU
    initial_W = numpy.asarray(
        numpy_rng.uniform(
            low=-4 * numpy.sqrt(6. / (n_hidden + n_visible)),
high=4 * numpy.sqrt(6. / (n_hidden + n_visible)),
            size=(n_visible, n_hidden)
        dtype=theano.config.floatX
    W = theano.shared(value=initial_W, name='W', borrow=True)
if not bvis:
    bvis = theano.shared(
        value=numpy.zeros(
            n_visible,
            dtype=theano.config.floatX
        borrow=True
    )
if not bhid:
    bhid = theano.shared(
        value=numpy.zeros(
            n_hidden,
            dtype=theano.config.floatX
        name='b',
        borrow=True
    )
self.W = W
# b corresponds to the bias of the hidden
self.b = bhid
# b_prime corresponds to the bias of the visible
self.b_prime = bvis
# tied weights, therefore W_prime is W transpose
self.W_prime = self.W.T
self.theano_rng = theano_rng
# if no input is given, generate a variable representing the input
if input is None:
    # we use a matrix because we expect a minibatch of several
    # examples, each example being a row
    self.x = T.dmatrix(name='input')
    self.x = input
self.params = [self.W, self.b, self.b_prime]
```

```
def get_corrupted_input(self, input, corruption_level):
     ""This function keeps ``1-corruption_level`` entries of the inputs the
    same and zero-out randomly selected subset of size ``coruption_level`
    Note: first argument of theano.rng.binomial is the shape(size) of
           random numbers that it should produce
           second argument is the number of trials
           third argument is the probability of success of any trial
            this will produce an array of 0s and 1s where 1 has a
            probability of 1 - ``corruption_level`` and 0 with
             `corruption_level``
            The binomial function return int64 data type by
            default. int64 multiplicated by the input
            type(floatX) always return float64. To keep all data
            in floatX when floatX is float32, we set the dtype of
            the binomial to floatX. As in our case the value of
            the binomial is always 0 or 1, this don't change the
            result. This is needed to allow the gpu to work
            correctly as it only support float32 for now.
    return self.theano_rng.binomial(size=input.shape, n=1,
                                    p=1 - corruption_level,
                                    dtype=theano.config.floatX) * input
def get_hidden_values(self, input):
        Computes the values of the hidden layer """
    return T.nnet.sigmoid(T.dot(input, self.W) + self.b)
def get_reconstructed_input(self, hidden):
       'Computes the reconstructed input given the values of the
    hidden Layer
    return T.nnet.sigmoid(T.dot(hidden, self.W_prime) + self.b_prime)
def get_cost_updates(self, corruption_level, learning_rate):
        This function computes the cost and the updates for one training
    step of the dA ""
    tilde x = self.get corrupted input(self.x, corruption level)
    y = self.get_hidden_values(tilde_x)
    z = self.get_reconstructed_input(y)
    # note : we sum over the size of a datapoint; if we are using
             minibatches, L will be a vector, with one entry per
             example in minibatch
    L = -T.sum(self.x * T.log(z) + (1 - self.x) * T.log(1 - z), axis=1)
    # note : L is now a vector, where each element is the
             cross-entropy cost of the reconstruction of the
             corresponding example of the minibatch. We need to
             compute the average of all these to get the cost of
             the minibatch
    cost = T.mean(L)
    # compute the gradients of the cost of the `dA` with respect
    # to its parameters
    gparams = T.grad(cost, self.params)
    # generate the list of updates
    updates = [
        (param, param - learning_rate * gparam)
        for param, gparam in zip(self.params, gparams)
    return (cost, updates)
```

Putting it All Together

It is easy now to construct an instance of our dA class and train it.

```
# allocate symbolic variables for the data
index = T.lscalar()  # index to a [mini]batch
x = T.matrix('x')  # the data is presented as rasterized images
```

```
# BUILDING THE MODEL CORRUPTION 30% #
######################################
rng = numpy.random.RandomState(123)
theano_rng = RandomStreams(rng.randint(2 ** 30))
da = dA(
    numpy_rng=rng,
    theano_rng=theano_rng,
    input=x.
    n_visible=28 * 28,
    n_hidden=500
cost, updates = da.get_cost_updates(
    corruption_level=0.3,
    learning_rate=learning_rate
train_da = theano.function(
    [index],
    cost.
    updates=updates,
    givens={
        x: train_set_x[index * batch_size: (index + 1) * batch_size]
)
start_time = timeit.default_timer()
###########
# TRAINING #
###########
# go through training epochs
for epoch in xrange(training_epochs):
    # go through trainng set
    c = []
    for batch index in xrange(n train batches):
        c.append(train_da(batch_index))
    print 'Training epoch %d, cost ' % epoch, numpy.mean(c)
end_time = timeit.default_timer()
training_time = (end_time - start_time)
print >> sys.stderr, ('The 30% corruption code for file ' +
                       os.path.split(__file__)[1] +
' ran for %.2fm' % (training_time / 60.))
```

In order to get a feeling of what the network learned we are going to plot the filters (defined by the weight matrix). Bear in mind, however, that this does not provide the entire story, since we neglect the biases and plot the weights up to a multiplicative constant (weights are converted to values between 0 and 1).

To plot our filters we will need the help of <u>tile_raster_images</u> (see <u>Plotting Samples and Filters</u>) so we urge the reader to study it. Also using the help of the Python Image Library, the following lines of code will save the filters as an image:

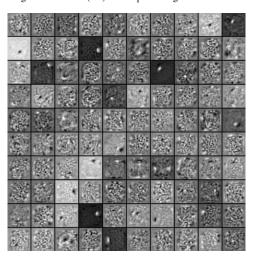
```
image = Image.fromarray(tile_raster_images(
    X=da.W.get_value(borrow=True).T,
    img_shape=(28, 28), tile_shape=(10, 10),
    tile_spacing=(1, 1)))
image.save('filters_corruption_30.png')
```

Running the Code

To run the code:

```
python dA.py
```

The resulted filters when we do not use any noise are:



The filters for 30 percent noise :

