Name:

| Problem | Points | Score |
| :--- | ---: | :--- |
| 1(a) | 10 |  |
| 1(b) | 10 |  |
| 1(c) | 10 |  |
| 1(d) | 10 |  |
| 2(a) | 15 |  |
| 2(b) | 10 |  |
| 3(a) | 10 |  |
| 3(b) | 10 |  |
| 3(c) | 15 |  |
| Total | 100 |  |

Notes:
(1) The exam is closed books and notes except for one double-sided sheet of notes.
(2) Please indicate clearly your answer to the problem.
(3) If I can't read or follow your solution, it is wrong and no partial credit will be awarded.

Problem No. 1: Consider the following models for a system that outputs sequences of the characters " $\$$ " and " $/ \mathrm{o}$ ". For these models, you must start in state 1 and end in state 2.
(a) Compute the probability that model A produced the sequence "\% $\%$ ".

(b) Which model most likely produced the sequence " $\% \$ \%$ ". Explain.
(c) Which state sequence most likely produced the sequence " $\% \$ \%$ ". What was the probability of that state sequence?
(d) Give at least two reasons why the probabilities in (a) and (c) differ.

## Problem No. 2:

(a) Given the two HMMs shown to the right, compute the most likely output sequence for Model A. What is the probability of this output sequence?

(b) Which model most likely produced the sequence "HTTT"? Show all work!

Problem No. 3: Consider a two-state model of a coin toss: $\boldsymbol{A}=\left[\begin{array}{cc}0.25 & 0.75 \\ 0.5 & 0.5\end{array}\right], \boldsymbol{B}=\left[\begin{array}{cc}0.5 & 0.5 \\ 0.5 & 0.5\end{array}\right]$, and $\boldsymbol{\pi}=\left[\begin{array}{ll}0.5 & 0.5\end{array}\right]$.
(a) Compute the probability that a sequence of two heads (e.g., HH) can be observed, or generated from this model.
(b) What is the most likely state sequence that produced this sequence of " HH "?
(c) Given a training sequence of "HTHT", reestimate the transition probabilities. Does this result make sense? Explain.

