**ECE 8527: Introduction to Machine Learning and Pattern Recognition**

# HW No. 3: Random Processes Revisited

1. Maximum Likelihood Estimates of the Mean:

Generate *N* points of a zero-mean unit variance white Gaussian noise 1D signal (e.g., *x[n]*). Plot the maximum likelihood estimate of the mean and variance as a function of *N*. Explain your results.

2. Autocorrelation and Covariance:

The autocorrelation function is defined as:



The covariance function is defined as:



The signal is assumed to be zero mean, though you can always remove the mean before computation.

Derive an expression for the autocorrelation and covariance of a zero mean Gaussian white noise signal. Explain the relationship between the autocorrelation function and the Fourier Transform spectrum (and what role phase plays in this). Describe the relationship between  and  and why this relationship holds. Explain how that impacts the shape of the autocorrelation function.

Next, derive an expression for the autocorrelation and covariance function for a sinewave. What is the rank of the covariance matrix? Why?

For these two signals, demonstrate that your computation of these functions on actual data produces a result close to the theoretical value. Show convergence of this estimate as *N*, the number of points, increases.

3. Correlation:

Apply your zero mean Gaussian white noise signal to a simple digital filter: . Derive an expression for an optimal way to compute the constant of this filter, *0.75*, from the output data in terms of the autocorrelation function, knowing that the input is zero mean Gaussian white noise. Demonstrate that you can estimate this parameter from actual data, and show that this estimate converges as *N* becomes large (be sure to plot the bias or error in the estimate as a function of *N*).

Repeat this for a second order filter: .

Is your estimate a maximum likelihood estimate? How would you prove that? How would you estimate these parameters using Bayesian techniques?

4. Stationarity:

Next, for the first filter in Problem 3, generate the output to Gaussian white noise, but this time varying the coefficient of the filter slowly. For example, let the coefficient of the filter vary according to the following equation:

, where  .

(The coefficient varies according to a *1 Hz* sinewave sampled at *100 Hz*.)

Analyze the signal in *N* points windows – starting at *m=0* and continuing for *m=N, 2N, 3N*, ...., compute an autocorrelation function:. Plot your estimate of *a* as a function of time, and a function of the number of points, *N*, used to compute the correlation function. Explain what happens? What is the optimal value of *N*?

5. Stability:

Consider a simple exponential signal: . Compute a *2x2* covariance matrix using a window whose first sample starts at *n=0*, *n=100*, *n=1000*. Do this for *a=0.99* and *a=1.01*. Explain your results.