Exam 2

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1 Problem 1

1.1 Part a

This problem considers two uniform probabilities for defined as shown in Eq 1 with equal prior. In the event that they are classified, we find the probability of error for two points randomly selected using Matlab.

$$P(X|\omega_1) = \begin{cases} 1 : 0 \leq x \leq 1 \\ 0 : elsewhere \end{cases} \tag{1}$$

$$P(X|\omega_2) = \begin{cases} 1 : 0.5 \leq x \leq 1.5 \\ 0 : elsewhere \end{cases} \tag{2}$$

From those distribution, the probability of error can be calculated as shown:

$$p(error_1) = p(\omega_1) \int_{R2} P(X|\omega_2) dx \tag{3}$$

1.2 Part b

In Matlab, the was calculated to be $P(error_1) = 0.25$, similarly $P(error_2) = 0.25$, $P(error_{total}) = 0.5$. This was simulated in Matlab, where two points were drawn randomly from the uniform distributions for 100 times. Through those 100 times, the error rate for both classes was equal 0.25.

$$\begin{aligned} p(error_1) &= 0.2724 \\ p(error_2) &= 0.3123 \\ p(error_sum) &= 0.5847 \end{aligned} \tag{4}$$

After more simulation, we see that this error increases as the data is increased but is not twice the Bayes error as the bound defines it to be.

$$P \leq 2 \cdot P_{Bayes} \tag{5}$$

2 Problem 2

2.1 a

Hidden Markof Models are the subject of this problem. The HMM shown in Figure 2 and 3 each have a known transition matrix and state matrix. It was observed that Model A outputted the following sequence %$%, the first question finds the probability of this sequence occurring.

$$P_{transmit} = \begin{vmatrix} & P(\$) & P(\%) \\ 1 & 0.25 & 0.75 \\ 2 & 0.25 & 0.75 \end{vmatrix}.$$

$$P_{state} = \begin{vmatrix} & 1 & 2 \\ 1 & 0.5 & 0.5 \\ 2 & 0 & 1 \end{vmatrix}.$$

The transmit matrix can be interpreted as $P(i, j)$ is the probability that $ or % is transmitted at the i states and the state probability denotes the probability that the model transitions from i to j. Finding the probability that the model outputs the following is calculated as follow:

$$\begin{aligned} & p\,(V = \%\&\%) \\ & = P(\%) \cdot P(\&) \cdot P(\&) \\ & = P(\%, 1) \cdot P(\&, 1) \cdot P(\&, 2) \end{aligned} \tag{6}$$

The solution to this is found through an iterative process,

$$\begin{aligned} & P(\%)P(1, 1)\alpha(0) + P(\&)P(1, 2)\alpha(1) + P(\%)P(2, 2)\alpha(2) \\ & \text{where } \alpha(i) \text{ equals to the previous probabilty of an event occuring} \\ & = 0.5 \cdot 0.75 \cdot 0.25 \cdot 0.5\alpha(1) \cdot 0.75 \cdot \alpha(2) \\ & = 0.0176 \end{aligned} \tag{7}$$

2.2 b

Observing Model 1 and Model 2, the question is asked, "which of the two models is more likely to occur?" This probability is dependent on the state probabilities and the transmit probabilities, if P_state for both models are the same, only the transmit probabilities will have matter. And since $P(\%, 1)$ and $P(\%, 2)$ have greater values within Model A than in Model B, we can concluded that Model A will have the higher probability of transmitting the sequence.

2.3 c

For the two model, there is only one possibility for observing such emitted output, this state sequence is $s_1 \to s_1 \to s_2$, and from the transmit matrix, the probability can be computed as $p(1, 1) \cdot p(1, 1) \cdot p(1, 2) = 0.25$

2.4 d

Finally, to conclude this exercise, we ask, what is the difference between the probability for the sequence obtained in **c** and that obtained in **a**. The probability obtained in **a** relates to the probability of the visible outputs, while the probability in **c** relates to the hidden states. These two are different, the two models could have gone through the same states but emit different visible outputs. Also the probability of those visible output depend highly on the states.