Name:

|  |  |  |
| --- | --- | --- |
| Problem | Points | Score |
| 1 | 20 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 20 |  |
| 5 | 20 |  |
| Total | 100 |  |

Notes:

1. The exam is closed books and notes except for one double-sided sheet of notes.
2. Please indicate clearly your answer to the problem.
3. If I can’t read or follow your solution, it is wrong and no partial credit will be awarded.

**Problem No. 1 (Chapter 2, No. 37(a)):** Consider a two-category classification problem in two dimensions with:



Calculate the Bayes decision boundary. (Note: don’t simply state the result, you must derive it).

**Problem No. 2 (Chapter 3, No. 38(a))**: Let *p***x**(**x**|*ω*i) be arbitrary densities with means ***µ****i* and covariances ***Σ****i* – not necessarily normal – for i = 1, 2. Let *y* = **w**t**x** be a projection, and let the induced one-dimensional densities p(y|*ωi*) have means *µi* and variances *σi*2. Show that the criterion function:



is maximized by:

.

**Problem No. 3 (Chapter 4, No. 13(a))**: Consider classifiers based on samples with priors *P*(ω1) = P(ω2) = 0.5 and the distributions:



What is the Bayes decision rule and the Bayes classification error? (Note: you must derive these.)

**Problem No. 4 (Chapter 6, No. 10):** Express the derivative of a sigmoid in terms of the sigmoid itself in the following case (for positive constant *a*): .

**Problem No. 5 (Chapter 8, No. 5):** Consider training a binary tree using the entropy impurity, and refer to Eqs. 1 and 5 (below) for the case of an arbitrary number of categories. Prove that the decrease in entropy impurity provided by a single yes/no query can never be greater than one bit.

