

Name: \_\_\_\_\_

Problem	Points	Score
1(a)	10	
1(b)	10	
1(c)	10	
2(a)	20	
2(b)	20	
3(a)	20	
3(b)	10	
Total	100	

Notes:

- (1) The exam is closed books and notes except for four double-sided sheet of notes.
- (2) Please indicate clearly your answer to the problem.
- (3) If I can't read or follow your solution, it is wrong and no partial credit will be awarded.

**Problem No. 1: Fast Fourier Transform**

(10 pts) (1a) A 5 kHz sinusoidal signal is sampled at 40 kHz and 16 periods of the signal are collected. What is the length  $N$  of the collected samples? Suppose an  $N$ -point DFT is performed. Then, at what DFT indices,  $k = 0, 1, \dots, N-1$ , do you expect to see any peaks in the DFT spectrum? In general, how is the number of periods contained in the  $N$  samples related to the DFT index at which you get a peak?

(10 pts) (1b) Suppose the frequency of the sinewave was changed to 5.12493847 kHz. Describe what happens to the spectrum that you observe using the same parameters as in (a).

(10 pts) (1c) How might you recover the exact frequency of the sinewave in the most computationally efficient manner possible?

**Problem No. 2: Filter Design**

(20 pts) (2a) Consider a simple averaging filter in which the output is computed three ways:

$$y[n] = x[n + 1] + x[n] + x[n - 1]$$

$$y[n] = x[n] + x[n - 1] + x[n - 2]$$

$$y[n] = 0.9 * y[n - 1] + x[n]$$

Compare and contrast these three filters in terms of their ability to accurately estimate the average of a time-varying signal.

(20 pts) (2b) Determine the (a) symmetric and (b) antisymmetric linear-phase FIR filter that has the shortest possible length and has at least one zero at the location  $z = 0.5j$ .

**Problem No. 3:** Design a sample rate conversion system that reduces the sample rate from 44.1 kHz to 8 kHz for music originally recorded at 44.1 kHz.

(20 pts) (a) Discuss the computational and filter design issues associated with this application, paying special attention to distortion and aliasing.

(10 pts) (b) Suppose this system was designed for voice instead of music. How might that influence your design from a computational point of view? Note that voice transmitted with a sample rate of 8 kHz has been perfectly acceptable for many years in the telephone network.

(10 pts) (3c) Will these properties hold if  $h[n]$  is an IIR filter? Explain the significance of your findings.