Lecture 35

"white noise" ⇒ flat spectrum

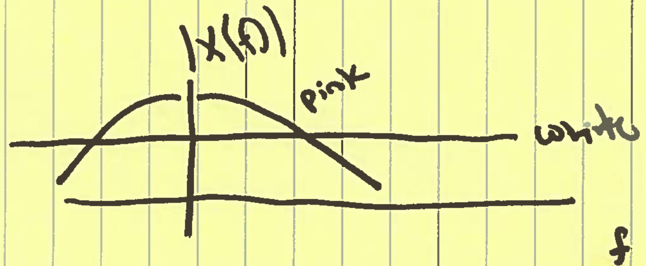
Gaussian noise ⇒ pdf of the amplitude is Gaussian

Laplacian ⇒ laplacian pdf

Gaussian white noise ⇒ pdf amplitude is Gaussian spectrum is white

Gaussian pink noise ⇒ Gaussian pdf spectrum is not flat

Laplacian white noise ⇒ Laplacian pdf white spectrum



Consider:

$$y(n) = x(n) + w(n)$$

All signals have noise!

$$E[x(n)] = ?$$

$$E[y(n)] = E[x(n) + w(n)]$$

$$= E[x(n)] + E[w(n)]$$

$$= \mu_x + \mu_w$$

Mitigate the effects of noise?

Draw lots of samples and average:

$$y_1(n) \Rightarrow E[y_1(n)]$$

$$y_2(n) \Rightarrow E[y_2(n)]$$

$$\vdots$$

$$y_p(n) \Rightarrow E[y_p(n)]$$

$$E\left[E[y_1(n)] + E[y_2(n)] + \dots\right] =$$

$$E[x(n)] + \cancel{E[w(n)]}^{\,0}$$

assuming zero mean noise!

Similarly, for the autocorrelation:

$$R_{yy}(\tau) = E\left[y(n)\,y(n+\tau)\right] = E\left[[x(n)+w(n)][x(n+\tau)+w(n+\tau)]\right]$$

$$= E\left[x(n)x(n+\tau)\right] + E\left[x(n)w(n+\tau)\right] + E\left[w(n)x(n+\tau)\right] + E\left[w(n)w(n+\tau)\right]$$

$$= R_{xx}(\tau) + 0 + 0 + R_{ww}(\tau)$$

if $x(n)$ is uncorrelated with the noise

also,

$$R_{ww}(\tau) = \mathcal{F}^{-1}\left\{S_w(f)\right\}$$

$$= \sigma^2 \delta(\tau)$$

$$R_{yy}(\tau) = R_{xx}(\tau) + \sigma^2 \delta(\tau)$$

- when noise is white

Note $R_{yy}(0) = E\left[y(n)y(n)\right] = E\left[y^2(n)\right]$

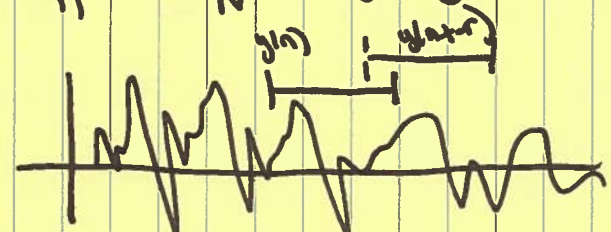
$= \text{Energy of } y(n)$

Energy of $y(n)$ is biased by the noise energy.

$$R_{yy}(\tau) = R_{xx}(\tau) + \sigma^2\delta(\tau)$$

autocorrelation removes the effects of noise for nonzero lags. (values of τ)

$$R_{yy}(\tau) = \frac{1}{N}\sum y(n)\,y(n+\tau)$$



Note:

$$R_{yy}(0) = \frac{1}{N}\sum y(n)\,y(n) = \text{energy!}$$

③

$y(n) = A \sin(\omega_0 t + \theta)$

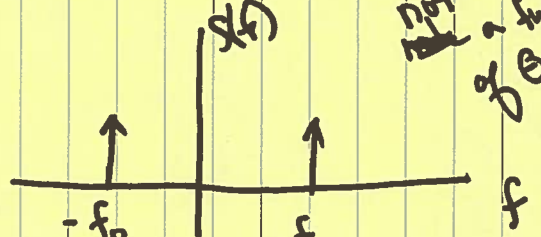
$R_{yy}(\tau) =$

Recall $R_{yy}(\tau) \Leftrightarrow S_y(f) = |Y(f)|^2$

Autocorrelation ⟺ Power Spectral Density

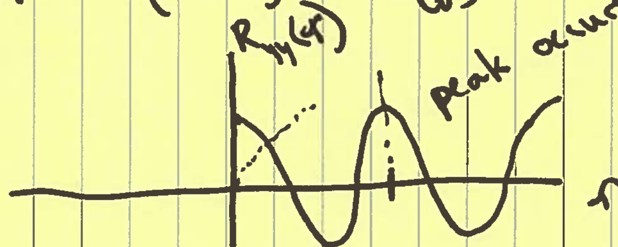
what is $S(f)$ for $A \sin(\omega_0 t + \theta)$

not a function of θ



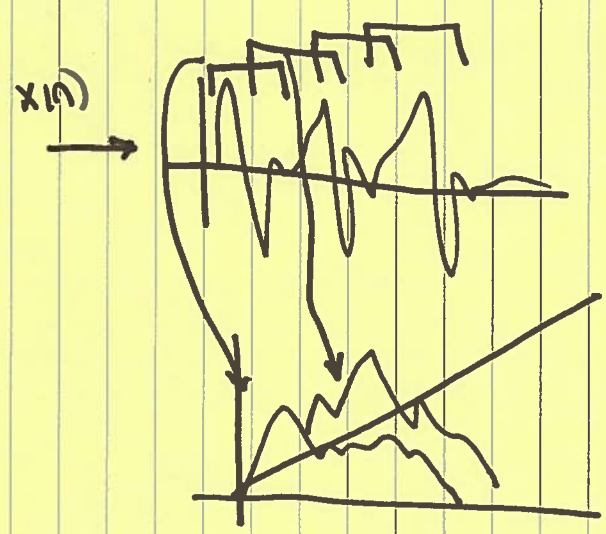
$R_{yy}(\tau) = \mathcal{F}^{-1}\{S(f)\} = A' \cancel{\sin}\ \cos(\omega_0 \tau)$

peak occurs at $\tau = 1/f_0$



signal → Autocor → frequency

"pick peak"



averaging spectra ⟹ improves our estimate