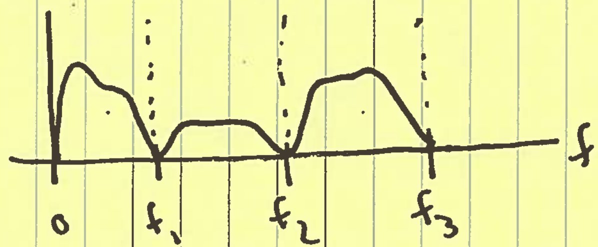
Lecture 33

Multirate signal processing



$[0, f_1]$: slowly-varying

$[f_1, f_2]$: not that ~~cr~~ critical

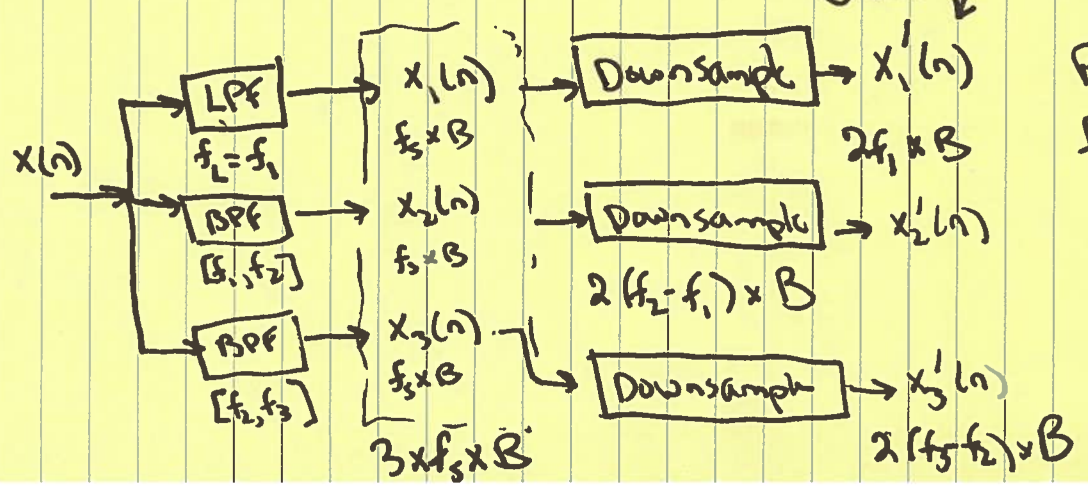
$[f_2, f_3]$: slowly-varying

The sampling theorem states: $f_s > 2f_3$

Analysis of quantization error: SNR $\propto$ 6B (B bits)

overall # of bits: $f_s B$

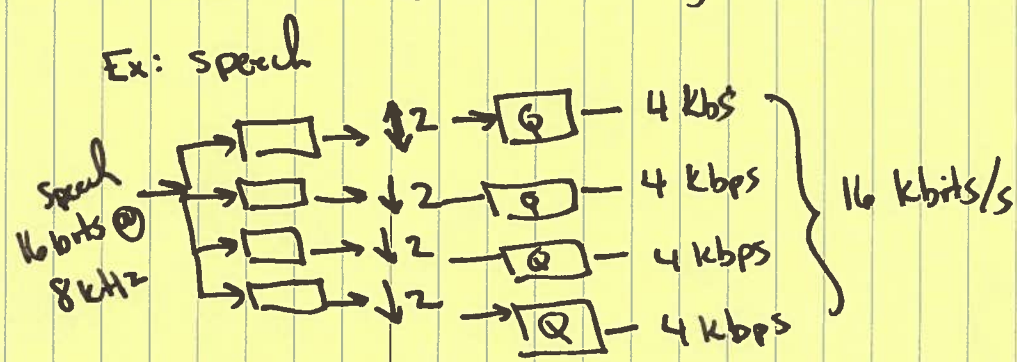
Can we do better?



Taken a signal, $x(n)$, sampled at $2f_3$, and converted it to 3 signals sampled at 3 different sample rates.

→ multirate signal processing

Ex: speech



(0, 500 Hz]

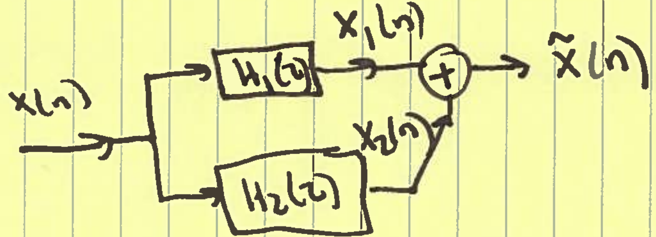
(500 to 1000 Hz)

(1000 Hz to 2000 Hz)

(2000 Hz to 4000 Hz)

"sub-band coding"

Potential for frequency dependent quantization

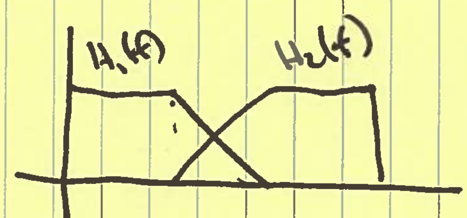


Requirement: $\tilde{x}(n) \simeq x(n)$

How do we achieve this?

· infinitely sharp
⇒ high order filters
⇒ expensive!

More reasonable approach:



composite frequency response is flat
$\Rightarrow \tilde{x}(n) = x(n)$

This problem has been extensively studied

The result was a filter structure known as Quadrature mirror filters (QMF).

Note: computationally efficient.

$$x_1(n) = x(n) * h_1(n)$$

$$= \sum_{m=0}^{M-1} x(m)\, h(n-m)$$

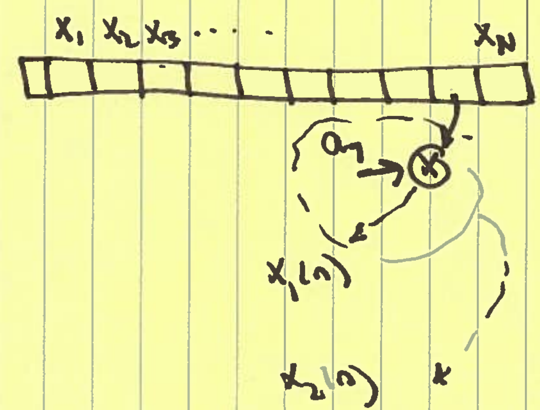
$$= \sum_{m=0}^{M-1} x(m)\, h_1(n-m)$$

Similarly have

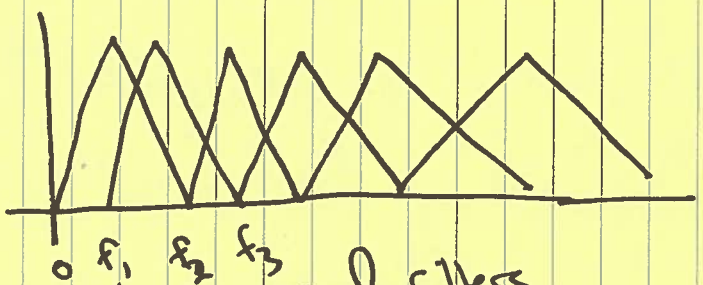
$$x_2(n) = \sum_{m=0}^{M-1} x(m)\, h_2(n-m)$$

Recall filter transforms:

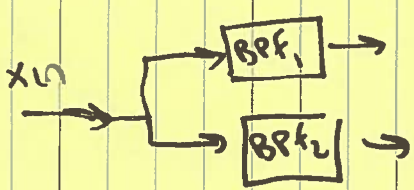
$$H_{LPF}(z) \rightarrow z \rightarrow \frac{1}{z} \rightarrow H_{HPF}(z)$$



Extensions to Pattern Recognition:



non-uniformly spaced filters



$y(n) = a\,y(n-1) + x^2(n)$

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Replaced both an FFT: