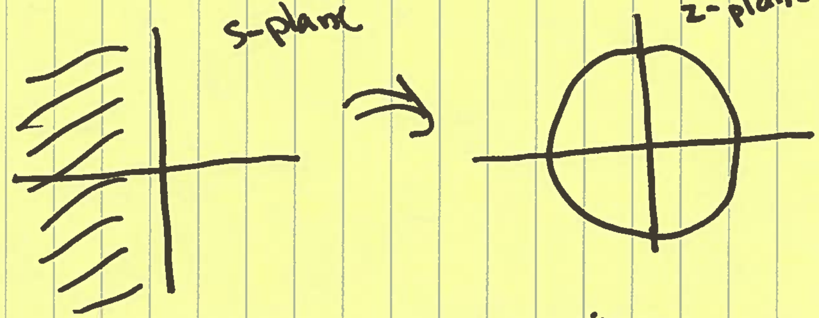
Lecture 21: IIR filter design

IIR filters are generally more compact (fewer coefficients) than FIR filters, but...

they have "nasty" phase responses.

We have a problem:



approach: (1) design an analog filter

(2) $s = f(z)$

(3) plot the frequency response of $H(z)$

unfortunately, the frequency axis gets "warped" the cutoff freq. in the analog domain shifts to a different frequency in the z-domain

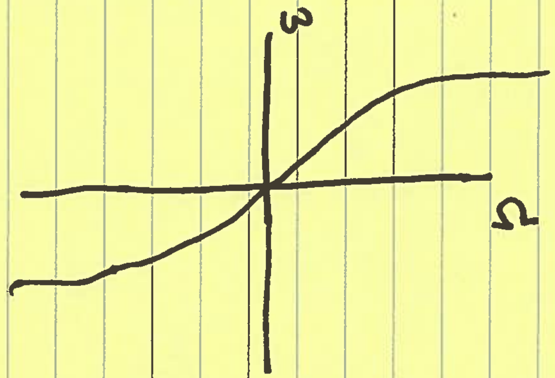
To do this transformation, we will use:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}} \quad \text{bilinear transformation}$$

$$j\Omega = \frac{1 - e^{-j\omega}}{1 + e^{-j\omega}}$$

simplifies to:

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$



strategy for design:

(1) specs

(2) map digital freq (cutoff freq.) to the corresponding analog freq.

(3) Design analog filter

(4) $s = \frac{1 - z^{-1}}{1 + z^{-1}} \Rightarrow H(z)$

converting filters:

$$s = \frac{1-z^{-1}}{1+z^{-1}} \quad \text{(lowpass)}$$

$$s = \frac{1+z^{-1}}{1-z^{-1}} \quad \text{(highpass)}$$

$$s = \frac{1-2cz^{-1}+z^{-2}}{1-z^{-2}} \quad \text{(bandpass)}$$

$$s = \frac{1-z^{-2}}{1-2cz^{-1}+z^{-2}} \quad \text{(bandstop)}$$

1st order Lowpass filters:

$$H_a(s) = \frac{\alpha}{s+\alpha}$$

$$H(z) = H_a(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}} = \frac{\alpha}{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+\alpha}$$

$$= \frac{\alpha(1+z^{-1})}{(1-z^{-1})+\alpha(1+z^{-1})} = \frac{b_0+b_1z^{-1}}{(1+\alpha)+(\alpha-1)z^{-1}}$$

$$= \frac{b_0+b_1z^{-1}}{1+a_1z^{-1}}$$

parameters of interest:

f_c in Hz

A_c in dB: attenuation at the cutoff freq.

assume passband gain is 1

Design equations?

$$H(s) = \frac{\alpha}{s+\alpha}$$

$s = j\Omega$

$$H(j\Omega)\Big|_{\Omega=f_c} = \frac{\alpha}{j\Omega+\alpha}\Big|_{\Omega=f_c} = G_c$$

$b_0 = \alpha$

$b_1 = \alpha$

$$\alpha = \frac{G_c}{\sqrt{1 - G_c^2}} \tan\left(\frac{\omega_c}{2}\right)$$

Specifies: $H(s) = \frac{\alpha}{s+\alpha}$

$$H(z) = b \frac{1+z^{-1}}{1-az^{-1}}$$

$$a = \frac{1-\alpha}{1+\alpha} \quad b = \frac{\alpha}{1+\alpha}$$

To get the equivalent highpass filter, use freq. transformations

Other ways to design filters?

thus far, focus is on cutoff freq. and attenuation

What about impulse response?