Lecture 22: Signal Averaging

Signal Averaging ***

$y(n) = \frac{1}{3}[x(n-1) + x(n) + x(n+1)]$

Impulse response?

$h(n) = \frac{1}{3}\delta(n-1) + \frac{1}{3}\delta(n) + \frac{1}{3}\delta(n+1)$

n	$\delta(n)$ $x(n)$	$\delta(n-1)$ $x(n-1)$	~~$\delta(n)$ $x(n)$~~	$\delta(n+1)$ $x(n+1)$	$y(n)$
-2	0	0	~~0~~	0	0
-1	0	0	~~0~~	1	1/3
0	1	0	~~1~~	0	1/3
1	0	1		0	1/3
2	0	0		0	0
3	0	0		0	0
4	0	0		0	0

FIR filter !!!

Averager (sliding window)

Low pass filter

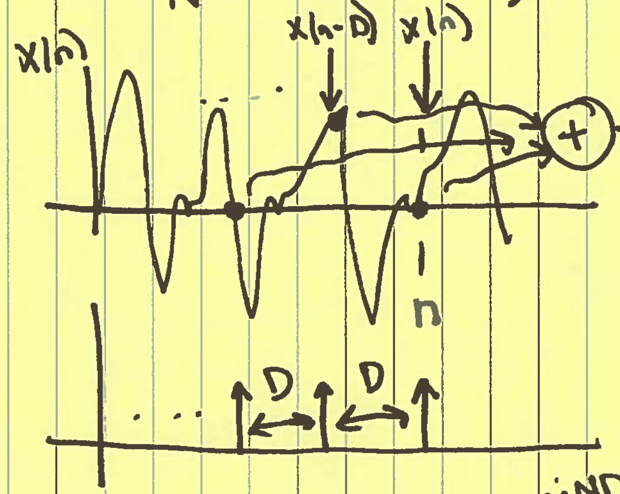
①

Signal averaging using comb filter

$$H(z) = \frac{1}{N}\left(1 + z^{-D} + z^{-2D} + \dots + z^{-(N-1)D}\right)$$

$$= \frac{1}{N}\,\frac{1 - z^{-ND}}{1 - z^{-D}}$$

$$h(n) = \frac{1}{N}\left[\delta(n) + \delta(n-D) + \delta(n-2D) + \dots\right]$$



$$H(\omega) = H(z)\Big|_{z=e^{j\omega}} = \frac{1}{N}\,\frac{1 - e^{-jND\omega}}{1 - e^{-jD\omega}}$$

$\Delta\omega = \frac{2\pi}{ND}$ $\quad \Delta f = \frac{1}{N T_D} = \frac{1}{NDT}$ $\quad T_D = DT$

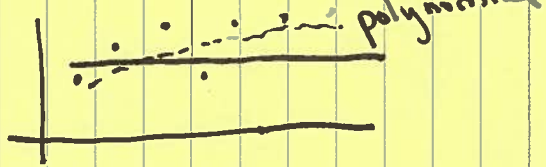
$$y(n) = \frac{1}{N}\left[x(n) + x(n-D) + \dots + x(n-(N-1)D)\right]$$

uses ND memory elements

Golay filters (Savitzky-Golay) SG

There are limits to how much smoothing an FIR filter with constant coefficients can do.

SG filters operate using polynomial smoothing



$\hat{x}_m = c_0$ constant

$\hat{x}_m = c_0 + c_1 m$ linear

$\hat{x}_m = c_0 + c_1 m + c_2 m^2$ quadratic

optimize c_0, c_1, c_2 based on data

Least squares approach:

$$\rightarrow J = \sum_{m=-2}^{2} e_m^2 = \sum_{m=-2}^{2} \left(x_m - (c_0 + c_1 m + c_2 m^2)\right)^2$$

minimize J with respect to c_0, c_1, c_2

approach: $\frac{\partial J}{\partial c_0} \cdots = 0$

Solution is well-known:

$$c = \underline{G}^T x$$

c: parameter vector (c_0, c_1, c_2)

$\underline{G}$: is a matrix:

$$\underline{G} = \underline{S}(\underline{S}^T \underline{S})^{-1}$$

$$\underline{S} : [s_0\ s_1\ s_2] = \begin{bmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

relates to basis functions

$s_0(m) = 1 \quad s_1(m) = m \quad s_2(m) = m^2$

implications:

$\underline{G}$ is a constant (precomputed)

$c = \underline{G}^T x$ data-dependent

$\Rightarrow$ time-varying filter

smoothed output:

$$\rightarrow \hat{x} = \underline{B}\, x$$

precomputed

$$\underline{B} = S(S^T S)^{-1} S^T$$