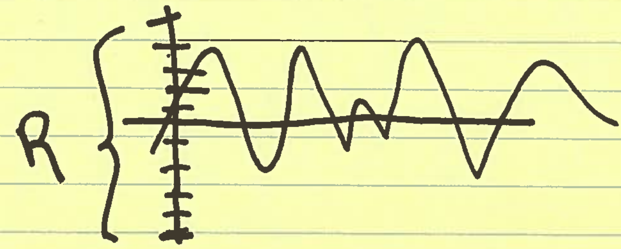
Quantization.



B bits
divide R uniformly

$$Q = \frac{R}{2^B} \qquad \frac{R}{Q} = 2^B$$

signal:

$$-\frac{R}{2} \le x_Q(nT) \le \frac{R}{2}$$

quantization error:

$$e(nT) = x_Q(nT) - x(nT)$$

$$-\frac{Q}{2} \le e \le \frac{Q}{2}$$

$$\bar{e} = \frac{1}{Q} \int_{-Q/2}^{Q/2} e\, p_e(e)\, de$$

assume uniformly distributed $p(\varepsilon) = \begin{cases} \frac{1}{Q} & -\frac{Q}{2} \le e \le \frac{Q}{2} \\ 0 & \text{elsewhere} \end{cases}$

$$\bar{e} = \frac{1}{Q} \int_{-Q/2}^{Q/2} e\, de = 0$$

$$E[e^2] = \int_{-Q/2}^{Q/2} e^2 p_e(e)\, de = \frac{1}{12}$$

$$e_{rms} = \sqrt{\frac{1}{12}} = \frac{1}{\sqrt{12}}$$

R = range   Q = step size

$$20\log_{10}\left(\frac{R}{Q}\right) = 20\log_{10}\left(2^B\right) = B \cdot 20\log_{10} 2$$

$$\text{SNR} = 2\log_{10}\left(\frac{R}{Q}\right) = 6B$$

every bit in quantization ≈ 6 dB improvement in SNR

high quality audio: 100 dB SNR

B = 16 !

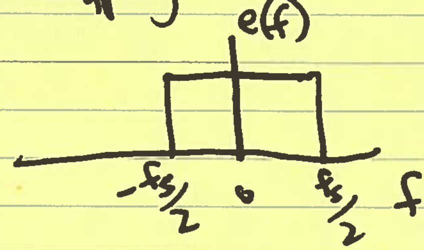
toll quality speech: ~ 40 dB SNR

8 bits for telephony ⇒ 48 dB SNR

$$x_q(n) = x(n) + e(n)$$

Oversampling and noise-shaping

random white noise ⇒



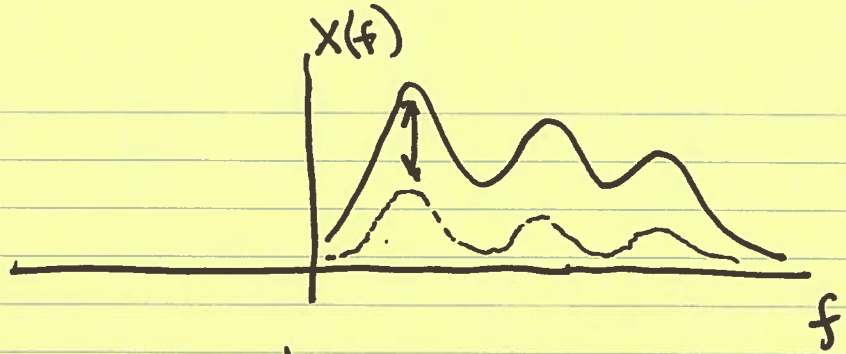
$$S_{ee}(f) = \frac{\sigma_e^2}{f_s}$$

for $\Delta f = f_b - f_a$

$$S_{ee}(f)\Delta f = \frac{\sigma_e^2 \Delta f}{f_s}$$

$H_{NS}(f)$ = shape the spectrum of the noise

$$S_{EE}(f) = |H_{NS}(f)|^2 S_{ee}(f) = \frac{\sigma_e^2}{f_s}|H_{NS}(f)|^2$$



f_s = sampling rate

f_s' = higher sampling rate

$$L = \frac{f_s'}{f_s}$$

$Q = R\,2^{-B}$ $\quad Q' = R\,2^{-B'}$ $\quad$ where $B' > B$

$$\sigma_e^2 = \frac{Q^2}{12} \qquad \sigma_e'^2 = \frac{Q'^2}{12}$$

If:

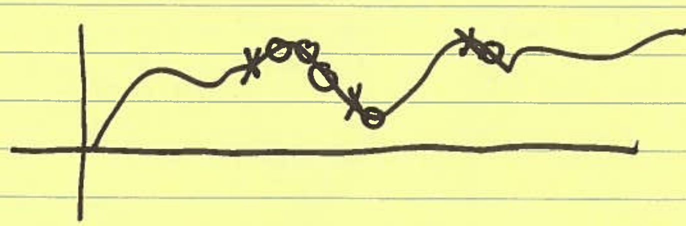
~~$\frac{\sigma_e^2}{12} = \frac{\sigma_e'^2}{12}$~~ $\quad \frac{\sigma_e^2}{f_s} = \frac{\sigma_e'^2}{f_s'}$

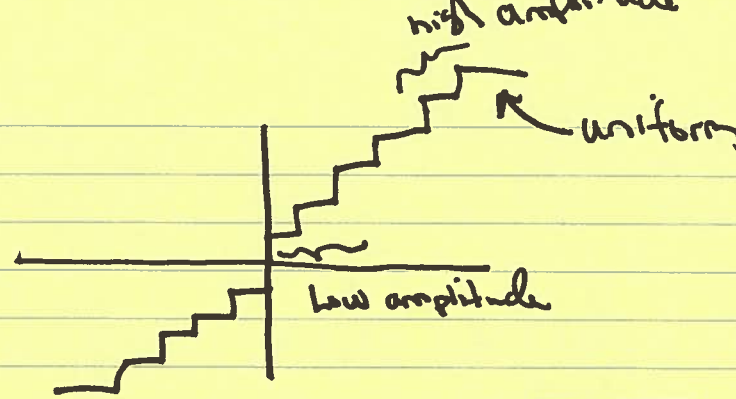
$$\sigma_e^2 = f_s \frac{\sigma_e'^2}{f_s'} = \frac{\sigma_e'^2}{L}$$

$$L = \frac{\sigma_e'^2}{\sigma_e} = 2^{2(B-B')} = 2^{2\Delta B}$$

$$\Delta B = 0.5 \log_2 L$$

every time we double L, we save $\frac{1}{2}$ bit — Oversampling





8-bit u-law ⇒ used phone network

12 bit linear ≅ 8-bit u-law

8 bits x 8 kHz = 64,000 bps