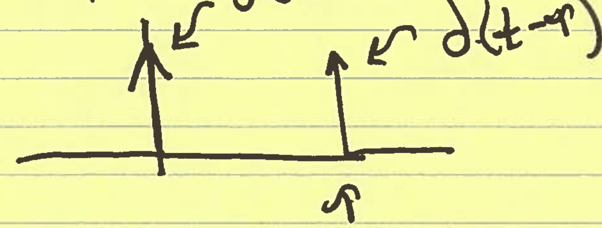
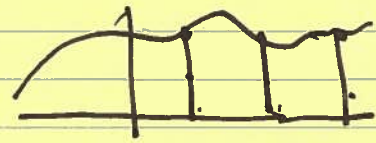
lecture 2

Spectrum of Sampled Signals: $\delta(t)$

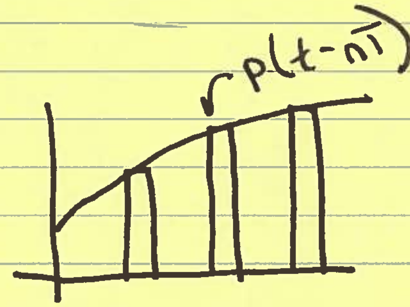
$\delta(t) \Rightarrow$



$$\hat{x}(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

in practice:




$$x_{flat}(t) = \sum_{n=-\infty}^{\infty} x(nT)\,p(t-nT)$$

Spectrum:

$$\hat{X}(f) = \int_{-\infty}^{\infty} \hat{x}(t)\, e^{-j2\pi f t}\, dt$$

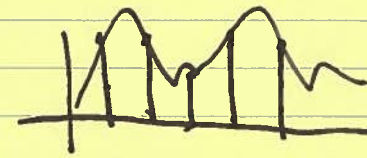
$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)\, e^{-2\pi f t}\, dt$$

$$= \sum_{n=-\infty}^{\infty} x(nT) \int_{-\infty}^{\infty} \delta(t-nT)\, e^{-2\pi f t}\, dt$$

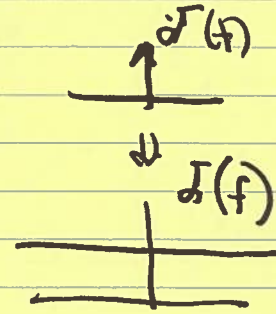
$$= \sum_{n=-\infty}^{\infty} x(nT)\, e^{-j2\pi f T n} \qquad T = \frac{1}{f_s}$$

Discrete-Time Fourier Transform

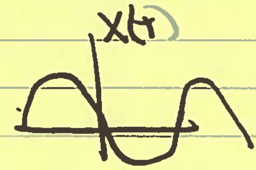
$$\hat{x}(t) = x(t)\underbrace{\sum_{n=-\infty}^{\infty} \delta(t-nT)}_{s(t)} \equiv x(t)s(t)$$



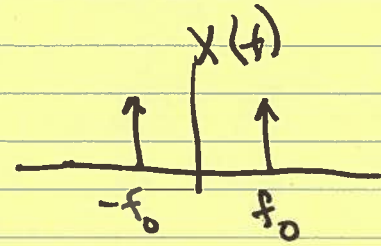
$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT) = \frac{1}{T}\sum_{m=-\infty}^{\infty} e^{j2\pi m f_s t}$$



Sinewave:



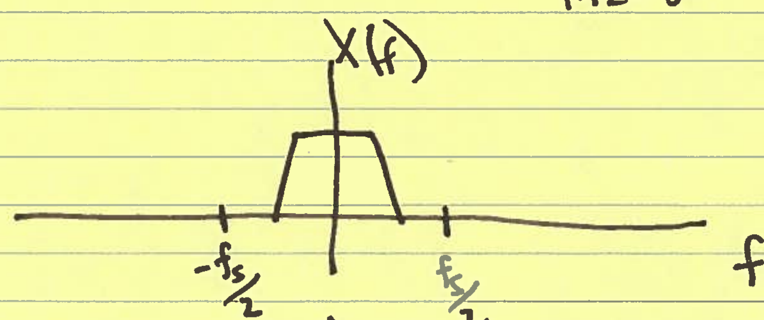

$\Rightarrow$

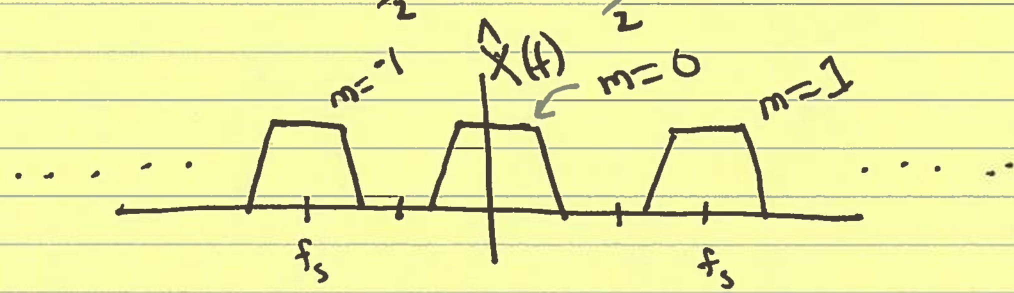



$$\hat{x}(t) = x(t)s(t) = \frac{1}{T}\sum_{m=-\infty}^{\infty} x(t)\, e^{j2\pi m f_s t}$$

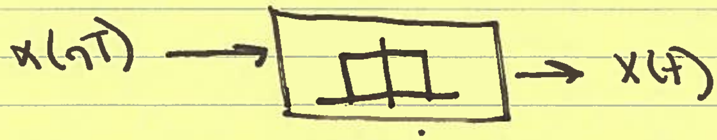
apply modulation theorem

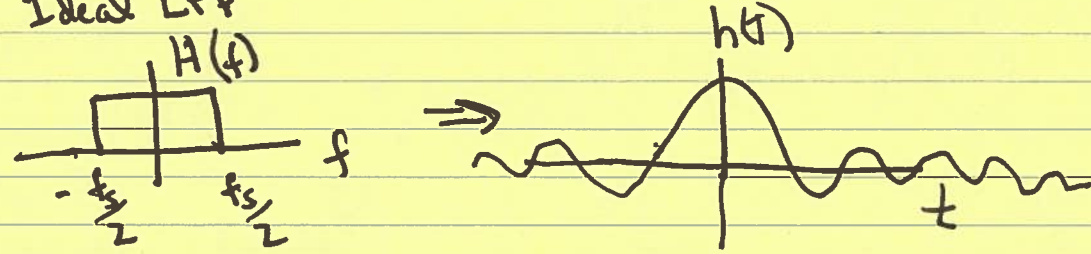
$$\hat{X}(f) = \mathcal{F}\{\hat{x}(t)\} = \frac{1}{T}\sum_{m=-\infty}^{\infty} X(f-mf_s)$$



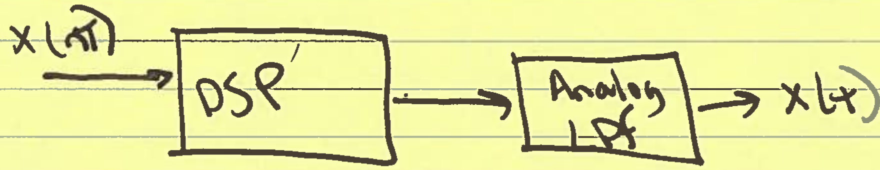
Reconstruction:

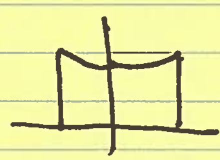


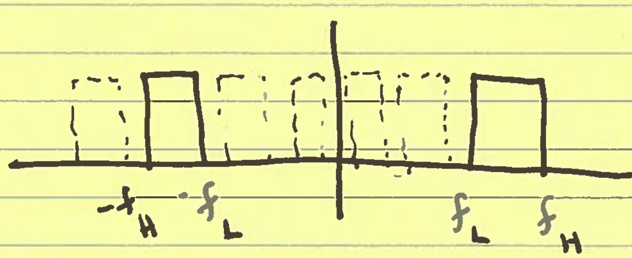

Can we build an ideal LPF? No!

Use an approximation:



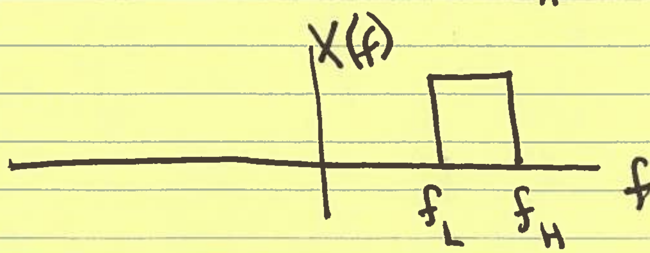



Bandlimited Sampling Theorem:




$f_s \geq 2f_H$

or $2(f_H - f_L)$




Complex!