Lecture 21

Noise reduction

$$x(n) = s(n) + v(n)$$

can we remove $v(n)$?
under what conditions?

if $s(n)$ + $v(n)$ occupy different places in the spectrum, simple filtering can be used

$$y(n) = y_s(n) + y_v(n)$$
$$= s(n) \circledast h(n) + v(n) \circledast h(n)$$

Figures of merit:

① Signal to Noise Ratio (SNR)

$$20\log_{10} \frac{E_S}{E_N}$$

② Noise Reduction Ratio (NRR)

To remove noise, use a filter that has the following properties:

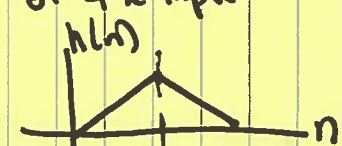
$$|H(f)| = 1$$

$$\angle H(f) = \alpha f$$

linear phase filter

$$y_s(n) = \int_{-\pi}^{\pi} Y_s(\omega)\, e^{j\omega n} \frac{d\omega}{2\pi}$$
$$= \int_{-\pi}^{\pi} |H(\omega)|\, S(\omega)\, e^{j\omega(n-D)} \frac{d\omega}{2\pi}$$
$$= s(n-D)$$

if a filter has linear phase, the output is a delayed version of the input **



typical noise reduction filters:

Averaging is a form of low pass filtering!

1st order IIR:

$$H(z) = \frac{1}{1-az^{-1}} = \frac{1}{1-az^{-1}} = \frac{Y(z)}{X(z)}$$

$$Y(z)\left[1-az^{-1}\right] = X(z)$$

$$Y(z) = az^{-1}Y(z) + X(z)$$

$$y(n) = a\,y(n-1) + x(n)$$

Low-Pass

Not Linear Phase!

FIR averager:

$$H(z) = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3}$$

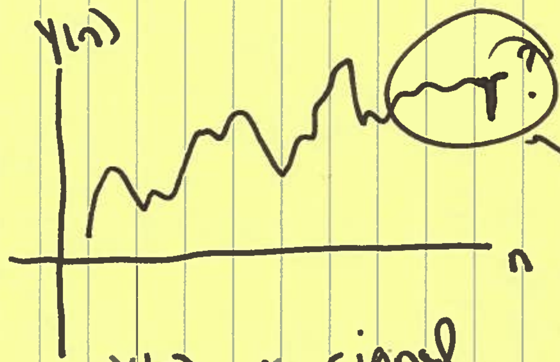
$$\frac{Y(z)}{X(z)} = h_0 + h_1 z^{-1} + h_2 z^{-2} + h_3 z^{-3}$$

$$Y(z) = h_0 X(z) + h_1 X(z) z^{-1} + h_2 X(z) z^{-2} + h_3 X(z) z^{-3}$$

$$y(n) = h_0 x(n) + h_1 x(n-1) + h_2 x(n-2) + h_3 x(n-3)$$

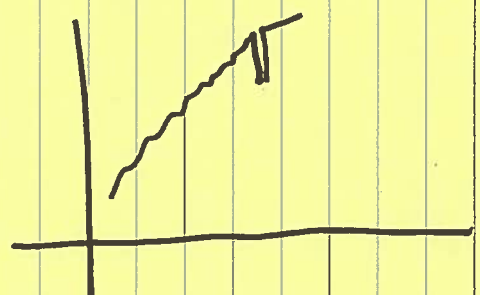
Ex: $y(n) = \frac{1}{4}\left[x(n) + x(n-1) + x(n-2) + x(n-3)\right]$ LPF!

Ex: $y(n) = \alpha y(n-1) + (1-\alpha) x(n)$



$x(n)$ is signal

$y(n)$ is the memory



Ex: $y(n) = \alpha y(n-1) + \left[\frac{1}{N}\left(\underbrace{x(n) - x(n-N)}\right)\right](1-\alpha)$