1

Lecture 8:

Time Invariance:

$x(n) \rightarrow \boxed{H} \rightarrow y(n)$

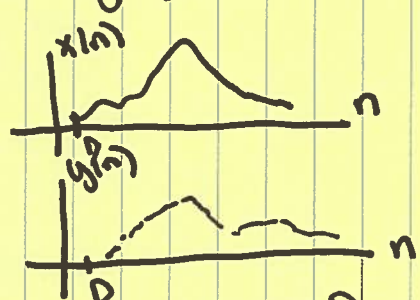
$x(n-D) \qquad y(n-D)$

Causality:

$x(n) \rightarrow \boxed{H} \rightarrow y(n)$

If $x(n)=0 \quad n<0$

$y(n)=0 \quad n<0$



$y(n)=0 \quad n<D$

$y(n)\neq 0 \quad n\geq D$

"the output does not precede the input"

IIR systems:

$$y(n)=\sum_{p=1}^{P} a_p x(n-p) + \sum_{q=1}^{Q} b_q y(n-q)$$

Ex: $y(n)= a_1 x(n-1)$ causal!

n	$x(n)$	$y(n)$
0	1	0
1	2	$a_1 = 2$
2	4	$a_1 x(1) = 4a_1$
3	6	$a_1 x(2) = 6a_1$
⋮		

$y(n)= a_1 x(n+1)$ non causal!

n	$x(n)$	$y(n)$
0	1	$2a_1$
1	2	$a_1 4$
2	4	$6a_1$
3	6	

$n=-1, \; y(-1)=a_1 x(0)$

Easy to spot:

$y(n)= a_1 x(n-1) + a_2 x(n) + a_3 x(n+1)$

Stability:

$$x(n) \rightarrow \boxed{H} \rightarrow y(n)$$

Bounded input / bounded output

$$\rightarrow \sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$y(n) = a_1 x(n) + a_2 x(n-2) + a_3 x(n-3)$$

$$x(n) = \delta(n)$$

$$y(n) = ?$$

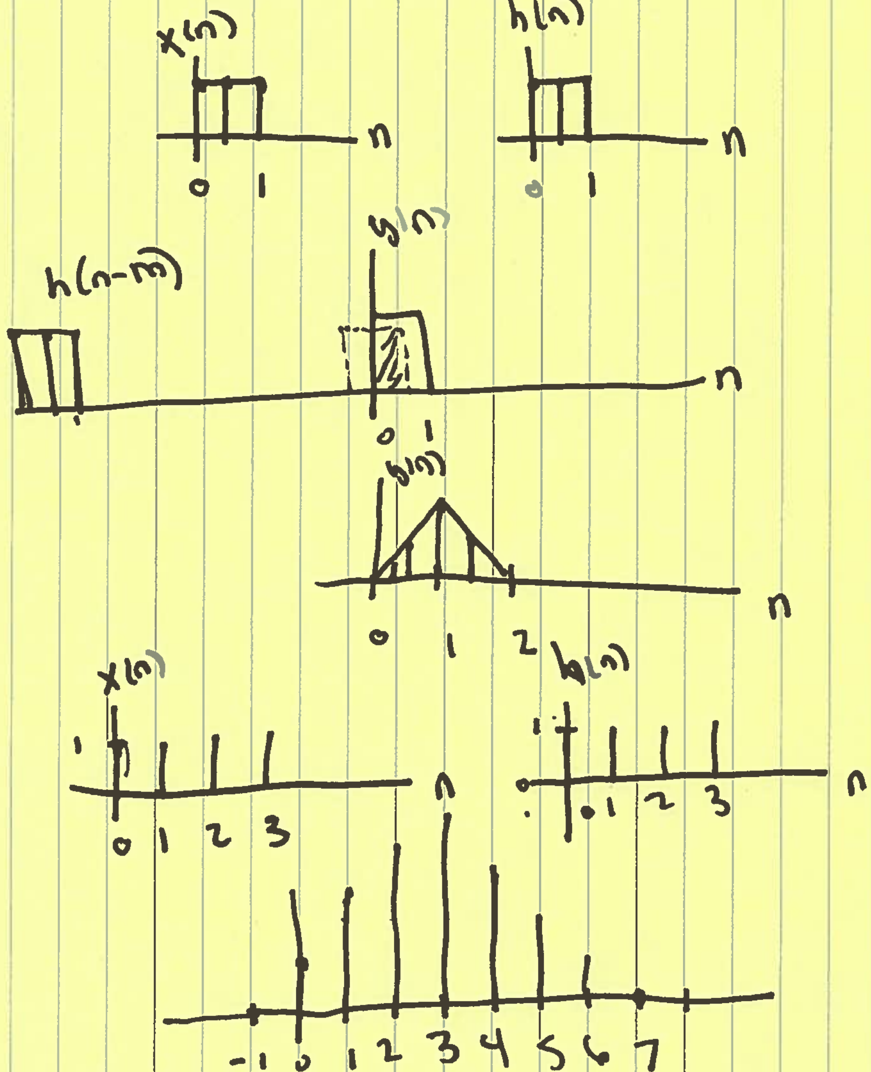
Finite Impulse Response (FIR) are stable.

$$y(n) = a_1 y(n-1) + x(n)$$

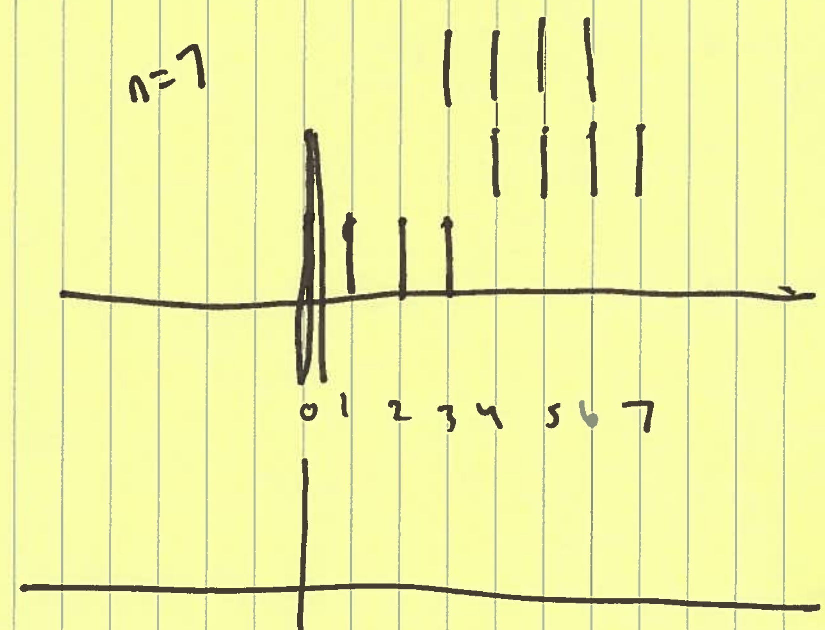
potentially unstable

Convolution:

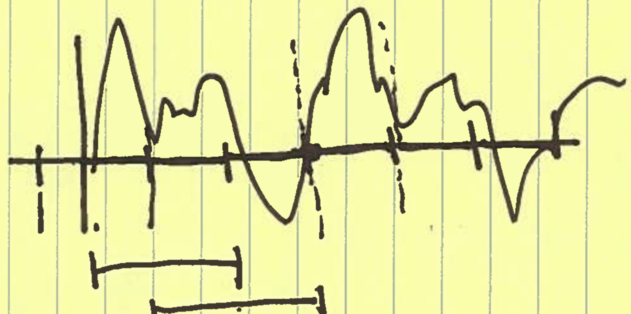
$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\,h(n-m)$$



n=7



Programming Implications



$y(n) = a x(n) + b x(n-1)$

$y(n) = a y(n-1) + b y(n-2) + c x(n) + d x(n-1)$

$x(n) = [0, 3, 7, 9, -1, \ldots]$