23. Solution:

$X \sim N(0,9)$ implies $X/3 \sim N(0,1)$, $y = 5x^2$ equals $y = 45(x/3)^2$. Define $z = (x/3)^2$, hence we have $Z \sim \chi_1^2$. The mth moments for Z can be written as

$$E[Z^m] = 2^m \frac{\Gamma(m+1/2)}{\Gamma(1/2)}$$

From which we can compute the first two moments as

$$E[z] = 2 \cdot \frac{\Gamma(1+1/2)}{\Gamma(1/2)} = 1$$

$$E[z^2] = 2^2 \cdot \frac{\Gamma(2+1/2)}{\Gamma(1/2)} = 3$$

Hence, we can compute the mean and variance of Y as

$$E[Y] = 45E[Z] = 45$$

$$\sigma_Y^2 = 45^2(E[Z^2] - E^2[Z]) = 45^2 \cdot (3-1) = 4050$$

26. (a) Solution:

$$\mu_w[n] = E[w[n]] = -4 \times \frac{1}{4} + 0 \times \frac{1}{4} + \frac{1}{2} \times 4 = 1$$

When $m = n$, the pmf of $w^2[n]$ is

$$P\{w^2[n] = 0\} = \frac{1}{4}, \quad P\{w^2[n] = 16\} = \frac{3}{4}$$

$$E[w^2[n]] = 0 \times \frac{1}{4} + 16 \times \frac{3}{4} = 12$$

When $m \neq n$, the pmf of $w[m]w[n]$ is

$$P\{w[m]w[n] = -16\} = \frac{1}{4}, \quad P\{w[m]w[n] = 0\} = \frac{7}{16}, \quad P\{w[m]w[n] = 16\} = \frac{5}{16}$$

$$E[w[m]w[n]] = -16 \times \frac{1}{4} + 0 \times \frac{7}{16} + 16 \times \frac{5}{16} = 1$$

Hence, the autocorrelation $r_w[m,n]$ is

$$r_w[m,n] = 11\delta[m-n] + 1$$

(b) Solution:

$$\mu_v[n] = E[v[n]] = \int_{-5}^{7} \frac{v}{12} dv = 1$$

When $m \neq n$

$$E[v[m]v[n]] = E[v[m]]E[v[n]] = 1$$

When $m = n$

$$E[v^2[n]] = \int_{-5}^{7} \frac{v^2}{12} dv = 13$$

Combining the two cases above, we conclude the autocorrelation $r_v[m,n]$ as

$$r_v[m,n] = 12\delta[m-n] + 1$$

(c) Solution:

$$r_{w,v}[m,n] = E[w[m]v[n]] = E[w[m]]E[v[n]] = 1$$

(d) Solution:

$$\mu_X[n] = E[x[n]] = E[w[n] + v[n-1]] = E[w[n]] + E[v[n-1]] = 2$$

(e) Proof:

$$\begin{aligned} r_x[m,n] &= E[x[m]x[n]] = E[(w[m]+v[m-1])(w[n]+v[n-1])] \\ &= E[w[m]w[n]] + E[w[m]v[n-1]] + E[v[m-1]w[n]] + E[v[m-1]v[n-1]] \\ &= (11\delta[m-n]+1) + 1 + 1 + (12\delta[m-n]+1) \\ &= 4 + 23\delta[m-n] \end{aligned}$$