```
% plot(t,x3)
% xlabel('t','fontsize',LFS)
% ylabel('x_3(t)','fontsize',LFS)
% sum(x3.*2/N)

%% Part (d):
t = linspace(0,0.01,N);
x4 = 4*sin(100*pi*t).*2.*cos(400*pi*t);
hf = figconfg('P0403','small');
plot(t,x4)
xlabel('t','fontsize',LFS)
ylabel('x_4(t)','fontsize',LFS)
sum(x4.*0.01/N)
```

4. (a) Solution:

The fundamental period of $x(t)$ is $T = 2$.

$$\int_0^2 \sin(3\pi t)dt = \int_0^2 \cos(8\pi t + \pi/3)dt = \int_0^2 \sin(3\pi t)\cos(8\pi t + \pi/3)dt = 0$$

$$\begin{aligned} P_{\text{av}} &= \frac{1}{T}\int_0^2 |x(t)|^2 dt \\ &= \frac{1}{2}\int_0^2 4dt + \frac{1}{2}\int_0^2 16\cos^2(3\pi t - \pi/2)dt + \frac{1}{2}\int_0^2 36\cos^2(8\pi t + \pi/3)dt \\ &= 4 + 8\int_0^2 \frac{1-\cos(6\pi t - \pi)}{2}dt + 18\int_0^2 \frac{1-\cos(16\pi t + 2\pi/3)}{2}dt \\ &= 30 \end{aligned}$$

(b) Solution:

$$\Omega_0 = 2\pi \cdot \frac{1}{T} = \pi$$

(c) Solution:

$$x(t) = 2\mathrm{e}^{\mathrm{j}0\pi t} + 2\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}\mathrm{e}^{j3\pi t} + 2\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}\mathrm{e}^{-j3\pi t} + 3\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{j8\pi t} + 3\mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}\mathrm{e}^{-j8\pi t}$$

$$c_0 = 2, c_3 = 2\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}}, c_{-3} = 2\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}, c_8 = 3\mathrm{e}^{\mathrm{j}\frac{\pi}{3}}, c_{-8} = 3\mathrm{e}^{-\mathrm{j}\frac{\pi}{3}}.$$

(d) Solution:

$$P_{\text{av}} = \sum_{k=-\infty}^{\infty} |c_k|^2 = 30$$

which verifies our computation in part (a).

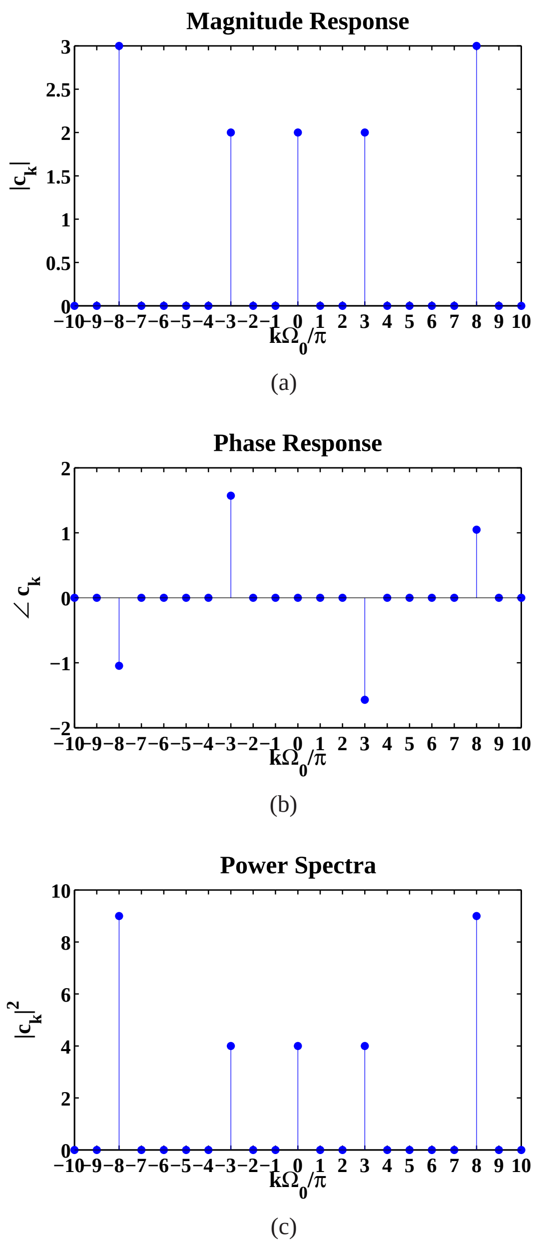

FIGURE 4.5: (a) Magnitude response of $x(t)$. (b) Phase response of $x(t)$. (c) Power spectra of $x(t)$.

9. (a) Solution:

$$X(\mathrm{j}2\pi F) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-\mathrm{j}2\pi Ft}dt = \int_{-\infty}^{\infty} \frac{2\sin 2\pi t}{2\pi t}\mathrm{e}^{-\mathrm{j}2\pi Ft}dt$$
$$= \begin{cases} 1, & -1 < F < 1 \\ 0, & \text{otherwise} \end{cases}$$

(b) Solution:

$$c_k = 4\int_{-\frac{1}{80}}^{\frac{1}{80}} 1\cdot \mathrm{e}^{-\mathrm{j}2\pi kF_0t}dt = 4\cdot \left.\frac{\mathrm{e}^{-\mathrm{j}2\pi kF_0t}}{-\mathrm{j}2\pi kF_0}\right|_{-\frac{1}{80}}^{\frac{1}{80}} = \frac{\sin\frac{\pi}{10}k}{\pi k}$$

(c) Solution:

$$X_s(\mathrm{j}2\pi F) = \sum_{k=-\infty}^{\infty} \frac{\sin\frac{\pi}{10}k}{\pi k}\cdot X[\mathrm{j}2\pi(F-4k)]$$

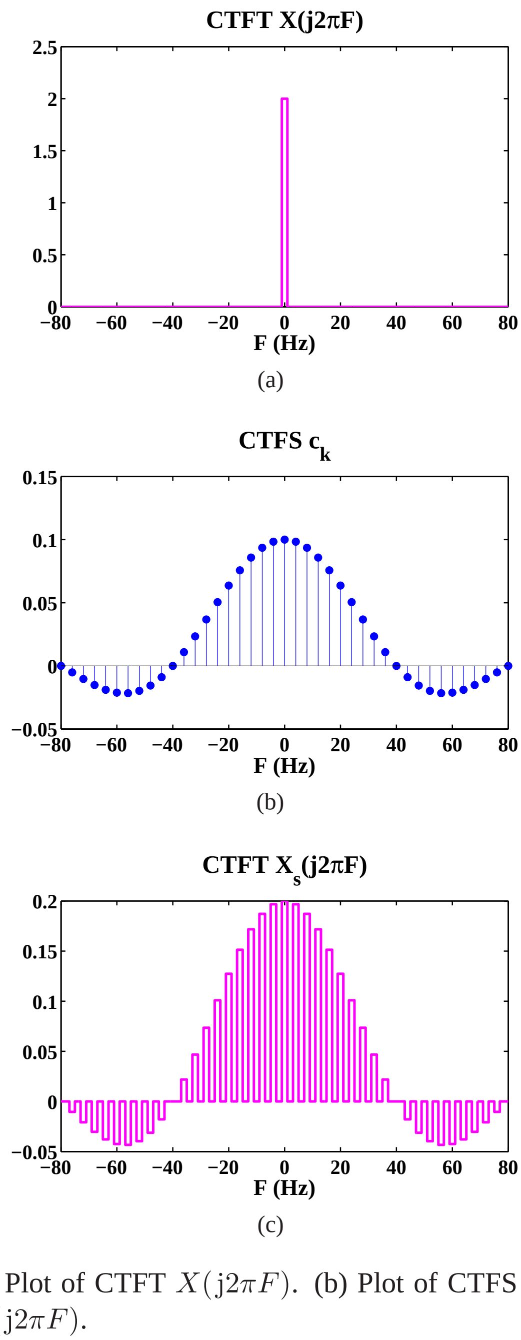

FIGURE 4.8: (a) Plot of CTFT $X(\mathrm{j}2\pi F)$. (b) Plot of CTFS coefficients c_k. (c) Plot of CTFT $X_s(\mathrm{j}2\pi F)$.

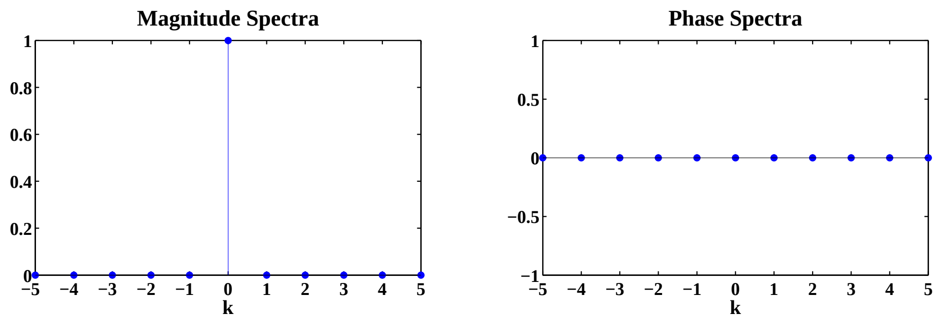

FIGURE 4.14: Magnitude and phase spectra of periodic sequence $x_6[n] = 1$ for all n.

12. Solution:
    (a)

$$X_1(\omega) = \frac{1}{1 - \mathrm{e}^{-\mathrm{j}\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega - 2k\pi)$$

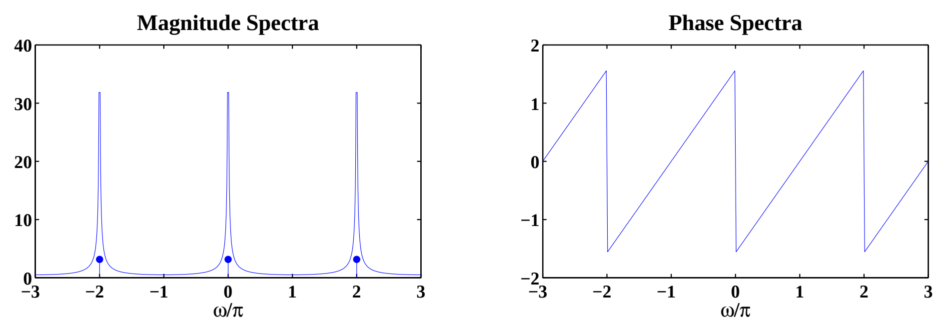

FIGURE 4.15: Magnitude and phase response for sequence $x_1[n] = u[n]$.

(b)

$$x_2[n] = \frac{1}{2}\left(\mathrm{e}^{\mathrm{j}\omega_0 n} + \mathrm{e}^{-\mathrm{j}\omega_0 n}\right) u[n]$$
$$= \frac{1/2}{1-\mathrm{e}^{-\mathrm{j}(\omega-\frac{\pi}{3})}} + \frac{1}{2}\sum_{k=-\infty}^{\infty} \pi\delta(\omega - \frac{\pi}{3} - 2k\pi)$$
$$\frac{1/2}{1-\mathrm{e}^{-\mathrm{j}(\omega+\frac{\pi}{3})}} + \frac{1}{2}\sum_{k=-\infty}^{\infty} \pi\delta(\omega + \frac{\pi}{3} - 2k\pi)$$

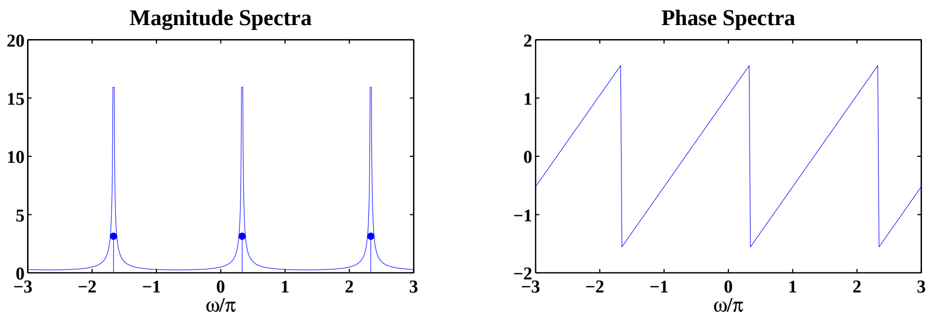

FIGURE 4.16: Magnitude and phase response for sequence $x_2[n] = \cos(\omega_0 n)u[n]$, $\omega_0 = \pi/3$.

13. (a) Solution:

$$x_1[n] = (1/2)^{|n|}\left(\frac{1}{2}\mathrm{e}^{\mathrm{j}\pi(n-1)/8} + \frac{1}{2}\mathrm{e}^{-\mathrm{j}\pi(n-1)/8}\right)$$
$$\text{DTFT}\left\{(1/2)^{|n|}\right\} = \sum_{n=-\infty}^{\infty}(1/2)^{|n|}\mathrm{e}^{-\mathrm{j}\omega n}$$
$$= \sum_{n=-\infty}^{-1}(1/2)^{-n}\mathrm{e}^{-\mathrm{j}\omega n} + 1 + \sum_{n=1}^{\infty}(1/2)^{n}\mathrm{e}^{-\mathrm{j}\omega n}$$
$$= \frac{3/2}{5/4-\cos\omega}$$
$$X_1(\omega) = \frac{1}{2}\mathrm{e}^{\mathrm{j}\pi/8}\frac{3/2}{5/4-\cos(\omega-\pi/8)} + \frac{1}{2}\mathrm{e}^{-\mathrm{j}\pi/8}\frac{3/2}{5/4-\cos(\omega+\pi/8)}$$

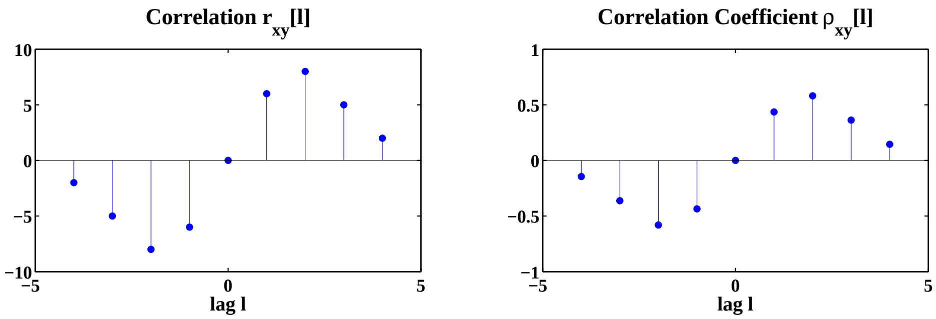

FIGURE 4.20: Plot of the correlation $r_{xy}[\ell]$ and correlation coefficient $\rho_{xy}[\ell]$ between the two signals.

18. (a) Solution:

$$
\begin{aligned}
r_{xy}[\ell] &= \sum_{n=-\infty}^{\infty} (0.9)^n u[n](0.9)^{n-\ell} u[n-\ell] \\
&= u[-\ell-1]\sum_{n=0}^{\infty}(0.9)^{2n-\ell} + u[\ell]\sum_{n=0}^{\ell}(0.9)^{\ell} \\
&= \frac{1}{1-0.9^2}\left(0.9^{-\ell}u[-\ell-1] + 0.9^{\ell}u[\ell]\right) \\
E_x &= \sum_{n=0}^{\infty} |(0.9)^n|^2 = \frac{1}{1-0.9^2}, \quad E_y = E_x \\
\rho_{xy}[\ell] &= \frac{r_{xy}[\ell]}{\sqrt{E_x}\sqrt{E_y}} = 0.9^{-\ell}u[-\ell-1] + 0.9^{\ell}u[\ell]
\end{aligned}
$$

(b) Solution:

$$
\begin{aligned}
r_{xy}[\ell] &= \sum_{n=-\infty}^{\infty} (0.9)^n u[n](0.9)^{-n+\ell} u[-n+\ell] \\
&= u[\ell]\sum_{n=\ell}^{\infty}(0.9)^{2n-\ell} = (\ell+1)(0.9)^{\ell}u[\ell] \\
E_x &= \sum_{n=0}^{\infty} |(0.9)^n|^2 = \frac{1}{1-0.9^2}, \quad E_y = E_x \\
\rho_{xy}[\ell] &= \frac{r_{xy}[\ell]}{\sqrt{E_x}\sqrt{E_y}} = (1-0.9)^2(\ell+1)(0.9)^{\ell}u[\ell]
\end{aligned}
$$

(c) Solution:

$$
\begin{aligned}
r_{xy}[\ell] &= \sum_{n=-\infty}^{\infty} (0.9)^n u[n] (0.9)^{n+5-\ell} u[n+5-\ell] \\
&= u[-\ell+4] \sum_{n=0}^{\infty} (0.9)^{2n+5-\ell} + u[\ell-5] \sum_{n=\ell-5}^{\infty} (0.9)^{2n+5-\ell} \\
&= \frac{1}{1-0.9^2} \left(0.9^{5-\ell} u[-\ell+4] + 0.9^{\ell-5} u[\ell-5] \right) \\
E_x &= \sum_{n=0}^{\infty} |(0.9)^n|^2 = \frac{1}{1-0.9^2}, \quad E_y = E_x \\
\rho_{xy}[\ell] &= \left(0.9^{5-\ell} u[-\ell+4] + 0.9^{\ell-5} u[\ell-5] \right)
\end{aligned}
$$

19.
```
function [rxy,l] = ccrs(x,nx,y,ny)
% P0419: Define function computing correlation rxy
%        between two finite length signals
% % Verification:
% nx = -2:2;
% ny = -2:2;
% x = [1 2 3 2 1];
% y = [2 1 0 -1 -2];
[rxy l] = conv0(x(:),nx(:),flipud(y(:)),sort(-ny));
```

Basic Problems

20. Solution:

$$x_1[n] = x_1[n + mN_1], \quad x_2[n] = x_1[n + mN_2], \quad m = 0, \pm 1, \pm 2, \ldots$$

$$x[n + N_1N_2] = x_1[n + N_1N_2] + x_2[n + N_1N_2] = x_1[n] + x_2[n] = x[n]$$

$x[n]$ is always periodic, and the fundamental period N is the least common multiple of N_1, N_2.

21. (a) Solution:

$$T_1 = \frac{2\pi}{3\pi} = \frac{2}{3}, \quad T_2 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$x_1(t)$ is aperiodic.

(b) Solution:

$$N = \frac{2\pi}{0.1\pi} = 20$$

$x_2[n]$ is periodic with fundamental period $N = 20$.

(c) Solution:

$$T_1 = \frac{2\pi}{3000\pi} = \frac{1}{1500}, \quad T_2 = \frac{2\pi}{2000\pi} = \frac{1}{1000}$$

$x_3(t)$ is periodic with fundamental period $T = \frac{1}{500}$.

(d) Solution:

$$N_1 = \frac{2\pi}{1/11} = 22\pi$$

$x_4[n]$ is aperiodic.

(e) Solution:

$$N_1 = \frac{2\pi}{\pi/5} = 10, \quad N_2 = \frac{2\pi}{\pi/6} = 12, \quad N = \frac{2\pi}{\pi/2} = 4$$

$x_5[n]$ is periodic with fundamental period $N = 60$.

22. (a) Solution:

$$x(t) = \cos(15\pi t) \implies x[n] = x(nT) = \cos(15\pi nT)$$

$$N = \frac{2\pi}{15\pi T} = \frac{2}{15T}$$

T is a rational number so that $x[n]$ is periodic.

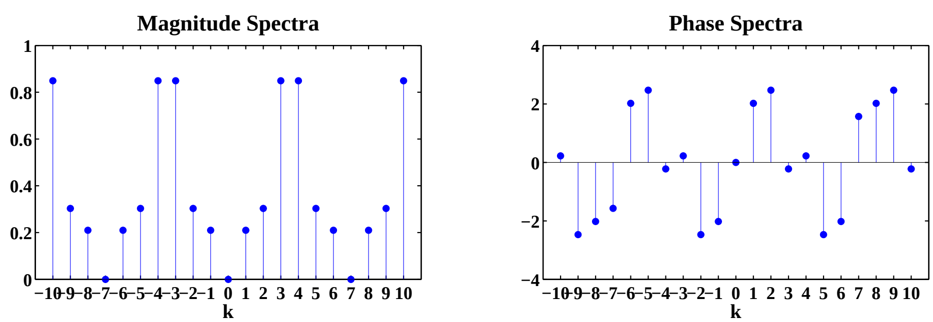

FIGURE 4.31: Magnitude and phase spectra of periodic sequence $x_5[n]$.

27. (a) Solution:

$$\frac{1}{N}\sum_{n=0}^{N-1} x[n-n_0]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = \frac{1}{N}\sum_{n=n_0}^{N-1+n_0} x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn}\cdot\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}kn_0} = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn_0}a_k$$

(b) Solution:

$$\frac{1}{N}\sum_{n=0}^{N-1}(x[n]-x[n-1])\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} = a_k - \mathrm{e}^{\mathrm{j}\frac{2\pi}{N}k}a_k$$

(c) Solution:

$$\begin{aligned}\frac{1}{N}\sum_{n=0}^{N-1}(-1)^n x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn} &= \frac{1}{N}\sum_{n=0}^{N-1}\mathrm{e}^{\mathrm{j}n\pi}x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}kn}\\ &= \frac{1}{N}\sum_{n=0}^{N-1}x[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}\left(k-\frac{N}{2}\right)n} = a_{k-\frac{N}{2}}\end{aligned}$$

(d) tba

28. (a) Solution:

$$y[n] = |x[n]|^2 = x[n]\cdot x^*[n]$$

35. (a) Proof:

$$r_{xy}[\ell] = \sum_{n=-\infty}^{\infty} x[n]y[n-\ell], \quad -\infty \le \ell \le \infty$$

$$\begin{aligned}
R_{xy}(\omega) &= \sum_{\ell=-\infty}^{\infty} r_{xy}[\ell]\mathrm{e}^{-\mathrm{j}\omega\ell} = \sum_{\ell=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} x[n]y[n-\ell]\mathrm{e}^{-\mathrm{j}\omega\ell} \\
&= \sum_{n=-\infty}^{\infty} x[n]\left(\sum_{\ell=-\infty}^{\infty} y[n-\ell]\mathrm{e}^{-\mathrm{j}\omega\ell}\right) \\
&= \sum_{n=-\infty}^{\infty} x[n]\mathrm{e}^{-\mathrm{j}\omega n}\left(\sum_{\ell=-\infty}^{\infty} y[n-\ell]\mathrm{e}^{-\mathrm{j}\omega(\ell-n)}\right) \\
&= \left(\sum_{n=-\infty}^{\infty} x[n]\mathrm{e}^{-\mathrm{j}\omega n}\right)\left(\sum_{m=-\infty}^{\infty} y[m]\mathrm{e}^{\mathrm{j}\omega m}\right) \\
&= X(\mathrm{e}^{\mathrm{j}\omega})Y\left(\mathrm{e}^{-\mathrm{j}\omega}\right)
\end{aligned}$$

(b) Proof:

$$R_x(\omega) = X(\mathrm{e}^{\mathrm{j}\omega})X\left(\mathrm{e}^{-\mathrm{j}\omega}\right)$$

Since $x[n]$ is real, $X\left(\mathrm{e}^{-\mathrm{j}\omega}\right) = X(\mathrm{e}^{\mathrm{j}\omega})^*$, hence

$$R_x(\omega) = |X(\mathrm{e}^{\mathrm{j}\omega})|^2$$

36. (a) See plot below.

(b) Comments:
The larger the delay D is, the smaller the correspondent $r_{xy}[\ell]$ will be. Hence, we can distinguish the delay D from the observation of $r_{xy}[\ell]$.

```
% P0436: Compute and plot correlation between x[n] and y[n]
% close all; clc
nx = -200:200;
xn = sin(0.2*pi*nx);
wn = randn(1,length(xn));
wn = sqrt(0.1)*wn;

D = 10;
% D = 20;
% D = 50;
```

44. (a) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{7}\sum_{n=-1}^{5} x_1[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}kn} \\
&= \frac{1}{7}\left(\mathrm{e}^{\mathrm{j}\frac{2\pi}{7}k} + 2 + 3\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k} + 3\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k\cdot 2} + 3\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k\cdot 3} + 2\mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k\cdot 4} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{7}k\cdot 5}\right)
\end{aligned}
$$

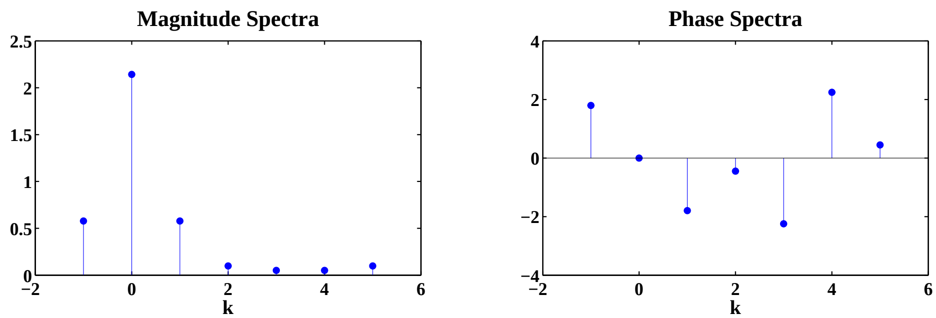

FIGURE 4.41: Magnitude and phase spectra of periodic sequence $x_1[n]$.

(b) Solution:

$$
\begin{aligned}
c_k &= \frac{1}{10}\sum_{n=-5}^{4} x_2[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{10}kn} \\
&= (-\frac{\mathrm{j}}{5})\sum_{n=1}^{4}\sin(0.2\pi n)\sin(0.2\pi kn)
\end{aligned}
$$

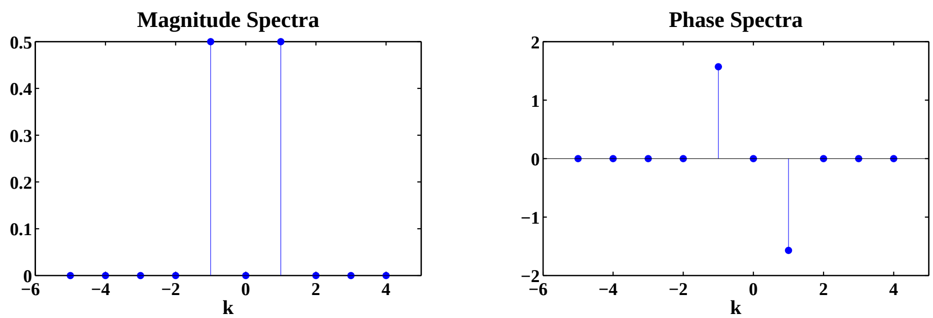

FIGURE 4.42: Magnitude and phase spectra of periodic sequence $x_2[n]$.

(c) Solution:

$$x_3[n] = \mathrm{e}^{\mathrm{j}2\pi n/7} + \mathrm{e}^{\mathrm{j}\pi n/3} + \mathrm{e}^{\mathrm{j}\pi n/7}$$
$$= \mathrm{e}^{\mathrm{j}\frac{2\pi}{42}n\cdot 6} + \mathrm{e}^{\mathrm{j}\frac{2\pi}{42}n\cdot 7} + \mathrm{e}^{\mathrm{j}\frac{2\pi}{42}n\cdot 3}$$

$$c_k = 1, \quad k = 3, 6, 7$$

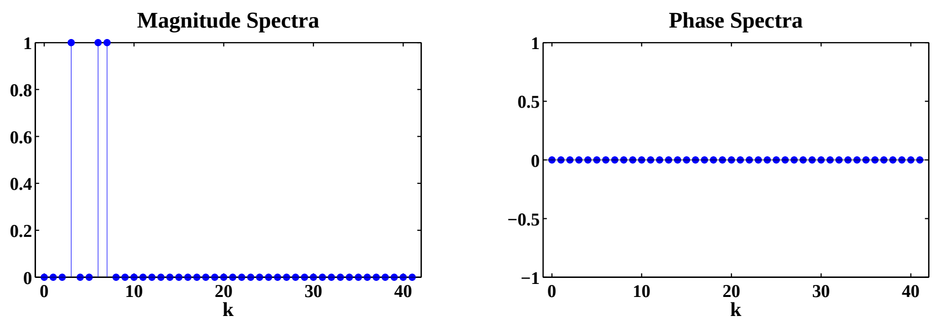

FIGURE 4.43: Magnitude and phase spectra of periodic sequence $x_3[n]$.

(d) Solution:

$$c_k = \frac{1}{8}\sum_{n=0}^{7} x_4[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{8}kn}$$
$$= \frac{1}{8}\left(1 + 2\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k} + 3\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 2} + 4\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 3} + 5\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 4} + 6\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 5} + 7\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 6} + 8\mathrm{e}^{-\mathrm{j}\frac{\pi}{4}k\cdot 7}\right)$$

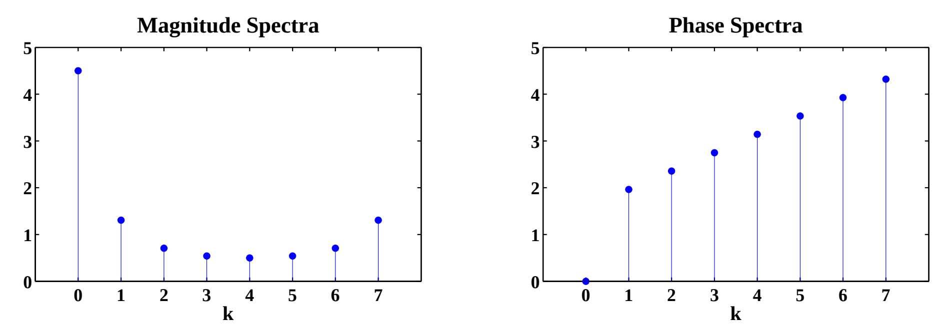

FIGURE 4.44: Magnitude and phase spectra of periodic sequence $x_4[n]$.

(e) Solution:

$$c_k = \frac{1}{2}\sum_{n=0}^{1} x_5[n]\mathrm{e}^{-\mathrm{j}\frac{2\pi}{2}kn}$$
$$= \frac{1}{2}(1\cdot 1 - 1\cdot \mathrm{e}^{-\mathrm{j}\pi k}) = \frac{1}{2}(1-\cos\pi k)$$

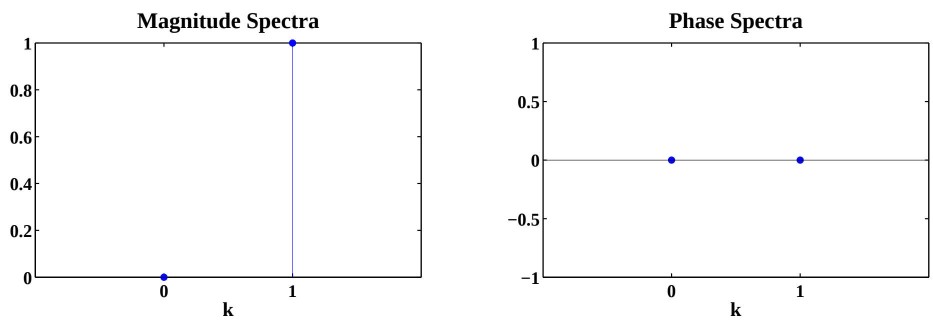

FIGURE 4.45: Magnitude and phase spectra of periodic sequence $x_5[n]$.

45. (a) Solution:

$$b_k = a_k(\mathrm{e}^{\mathrm{j}k\frac{2\pi}{N}} + 2 + \mathrm{e}^{-\mathrm{j}k\frac{2\pi}{N}}) = 2a_k(1+\cos(\frac{2\pi}{N}k))$$

(b) Solution:

$$\mathrm{e}^{-\mathrm{j}6\pi n/N}x[n-2] = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}n3}x[n-2]$$
$$b_k = a_{k+3}\mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}(k+3)2}$$

(c) Solution:

$$3\cos(2\pi 5n/N)x[-n] = \frac{3}{2}\left(\mathrm{e}^{\mathrm{j}\frac{2\pi}{N}n5} + \mathrm{e}^{-\mathrm{j}\frac{2\pi}{N}n5}\right)x[-n]$$
$$b_k = \frac{3}{2}a_{-(k-5)} + \frac{3}{2}a_{-(k+5)}$$

(d) Solution:

$$b_k = a_k + a_k^*$$

55. (a) Solution:

$$
\begin{aligned}
r_y[\ell] &= \sum_{n=-\infty}^{\infty} y[n]y[n-\ell] = \sum_{n=-\infty}^{\infty} (x[n]+ax[n-D])(x[n-\ell]+ax[n-D-\ell]) \\
&= \sum_{n=-\infty}^{\infty} \big(x[n]x[n-\ell] + x[n]\cdot a\cdot x[n-D-\ell] + a\cdot x[n-D]x[n-\ell] \\
&\quad + a^2\cdot x[n-D]x[n-D-\ell]\big) \\
&= (1+a^2)r_x[\ell] + a\cdot r_x[\ell+D] + a\cdot r_x[\ell-D]
\end{aligned}
$$

(b) See plot below.

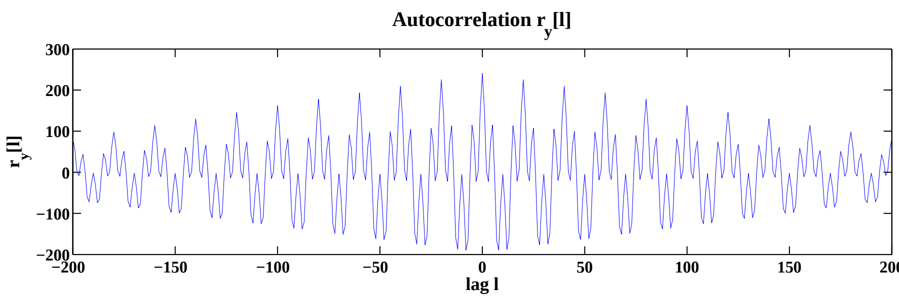

FIGURE 4.52: Plot of autocorrelation $r_y[\ell]$.

(c) tba

Review Problems

56. See book companion toolbox.

57. (a) Solution:

$$\begin{aligned}
c_k &= \int_{-0.5}^{0.5} \frac{1-4|t|}{2} \mathrm{e}^{-\mathrm{j}2\pi kt} dt \\
&= \int_{-0.5}^{0} \frac{1+4t}{2} \mathrm{e}^{-\mathrm{j}2\pi kt} dt + \int_{0}^{0.5} \frac{1-4t}{2} \mathrm{e}^{-\mathrm{j}2\pi kt} dt \\
&= \int_{0}^{0.5} (1-4t) \cos 2\pi kt dt \\
&= \frac{1-\cos \pi k}{\pi^2 k^2}
\end{aligned}$$

(b) See plot below.

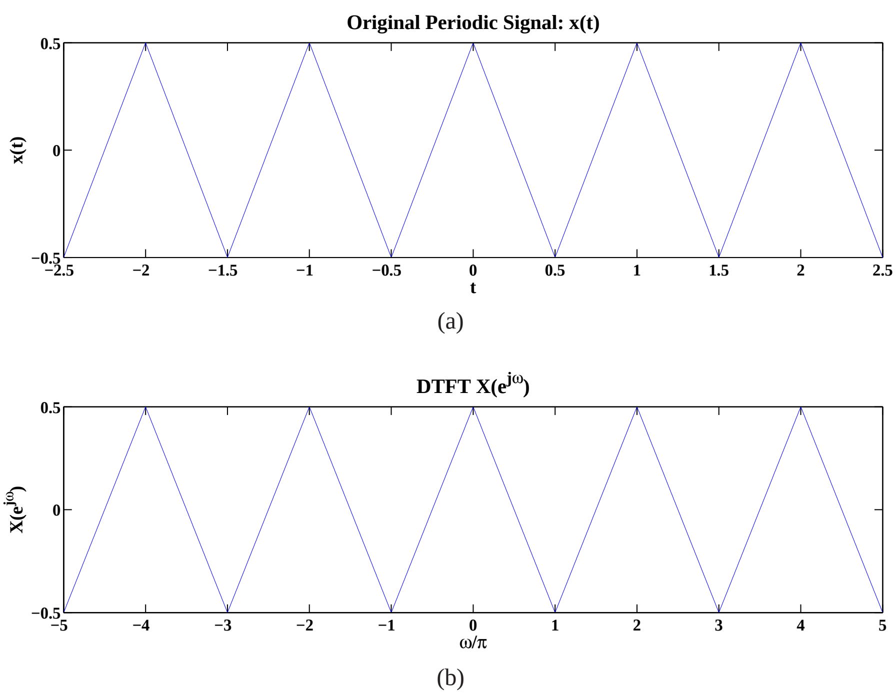

FIGURE 4.53: Plot of (a) original periodic signal $x(t)$ and (b) DTFT $C\big(\mathrm{e}^{\mathrm{j}\omega}\big)$.

(c) Solution:

$$x(t) = C\big(\mathrm{e}^{\mathrm{j}2\pi t}\big)$$

(d) tba