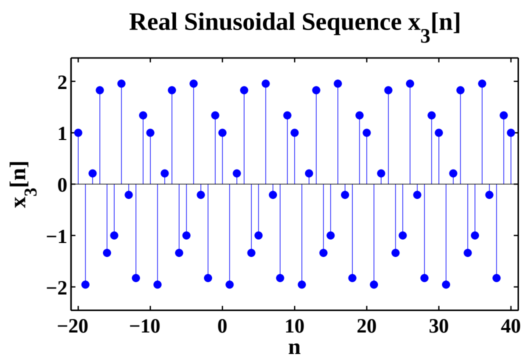

FIGURE 2.5: sinusoidal sequence $x_3[n] = 2\cos[2\pi(0.3)n + \pi/3]$.

2. MATLAB script:

```
% P0202: Illustrate the noncommutativity of folding and shifting
close all; clc
nx = 0:4; % specify the support
x = 5:-1:1; % specify sequence
n0 = 2;
% (a) First folding, then shifting
[y1 ny1] = fold(x,nx);
[y1 ny1] = shift(y1,ny1,-n0);
% (b) First shifting, then folding
[y2 ny2] = shift(x,nx,-n0);
[y2 ny2] = fold(y2,ny2);
% Plot
hf = figconfg('P0202');
xylimit = [min([nx(1),ny1(1),ny2(1)])-1,max([nx(end),ny1(end)...
    ,ny2(end)])+1,min(x)-1,max(x)+1];
subplot(3,1,1)
stem(nx,x,'fill')
axis(xylimit)
ylabel('x[n]','fontsize',LFS); title('x[n]','fontsize',TFS);
set(gca,'Xtick',xylimit(1):xylimit(2))
subplot(3,1,2)
stem(ny1,y1,'fill')
axis(xylimit)
ylabel('y_1[n]','fontsize',LFS);
```

```
title('y_1[n]: Folding and Shifting','fontsize',TFS)
set(gca,'Xtick',xylimit(1):xylimit(2))
subplot(3,1,3)
stem(ny2,y2,'fill')
axis(xylimit)
xlabel('n','fontsize',LFS); ylabel('y_2[n]','fontsize',LFS);
title('y_2[n]: Shifting and Folding','fontsize',TFS)
set(gca,'Xtick',xylimit(1):xylimit(2))
```

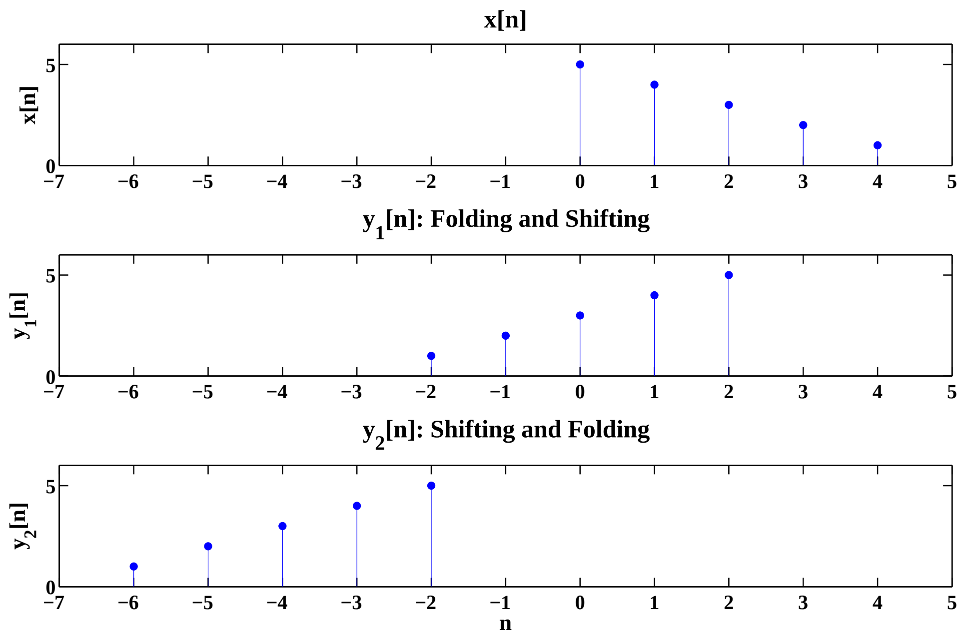

FIGURE 2.6: Illustrating noncommunativity of folding and shifting operations.

Comments:
From the plot, we can see $y_1[n]$ and $y_2[n]$ are different. Indeed, $y_1[n]$ represents the correct $x[2-n]$ signal while $y_2[n]$ represents signal $x[-n-2]$.

3. (a) $x[-n] = \{4, 4, 4, 4, \underset{\uparrow}{4}, 3, 2, 1, 0, -1\}$

$x[n-3] = \{-1, 0, \underset{\uparrow}{1}, 2, 3, 4, 4, 4, 4, 4\}$

$x[n+2] = \{-1, 0, 1, 2, 3, 4, 4, \underset{\uparrow}{4}, 4, 4\}$

```
n = -20:20; % support
w1 = 0.1; % angular frequency
x1 = cos(w1*n-pi/5);
% Part (c): Periodic
w2 = 0.1*pi; % angular frequency
x2 = cos(w2*n-pi/5);
%Plot
hf = figconfg('P0205');
xylimit = [n(1)-1,n(end)+1,min(x1)-0.5,max(x1)+0.5];
subplot(2,1,1)
stem(n,x1,'fill'); axis(xylimit)
xlabel('n','fontsize',LFS); ylabel('x_1[n]','fontsize',LFS);
title('Nonperiodic Sequence','fontsize',TFS);
% set(gca,'Xtick',xylimit(1):xylimit(2))
subplot(2,1,2)
stem(n,x2,'fill'); axis(xylimit)
xlabel('n','fontsize',LFS); ylabel('x_2[n]','fontsize',LFS);
title('Periodic Sequence','fontsize',TFS)
```

6. MATLAB script:

```
% P0206: Investigates the effect of downsampling using
%        audio file 'handel'
close all; clc
load('handel.mat')
n = 1:length(y);
% Part (a): original sampling rate
sound(y,Fs); pause(1)
% Part (b): downsampling by a factor of two
y_ds2_ind = mod(n,2)==1;
sound(y(y_ds2_ind),Fs/2); pause(1)
% Part (c): downsampling by a factor of four
y_ds4_ind = mod(n,4)==1;
sound(y(y_ds4_ind),Fs/4)
% save the sound file
wavwrite(y(y_ds4_ind),Fs/4,'handel_ds4')
```

7. Comments: The first system is NOT time-invariant but the second system is time invariant.

   MATLAB script:

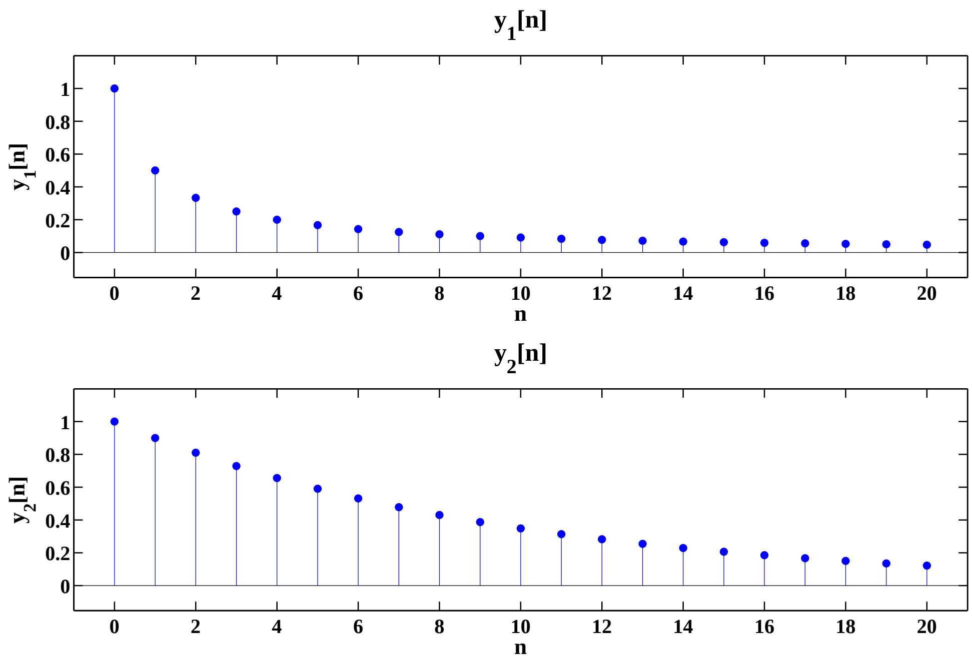

FIGURE 2.12: System responses with respect to input signal $x[n] = \delta[n]$.

```
% P0207: Compute and plot sequence defined by difference equations
close all; clc
n = 0:20; % define support
yi = 0; % zero initial condition
xn = delta(n(1),0,n(end))'; % input 1
% xn = delta(n(1),5,n(end))'; % input 2
% Compute sequence 1:
yn1 = zeros(1,length(n));
yn1(1) = n(1)/(n(1)+1)*yi+xn(1);
for ii = 2:length(n)
    yn1(ii) = n(ii)/(n(ii)+1)*yn1(ii-1)+xn(ii);
end
% Compute sequence 2:
yn2 = filter(1,[1,-0.9],xn);
%Plot
hf = figconfg('P0207');
```

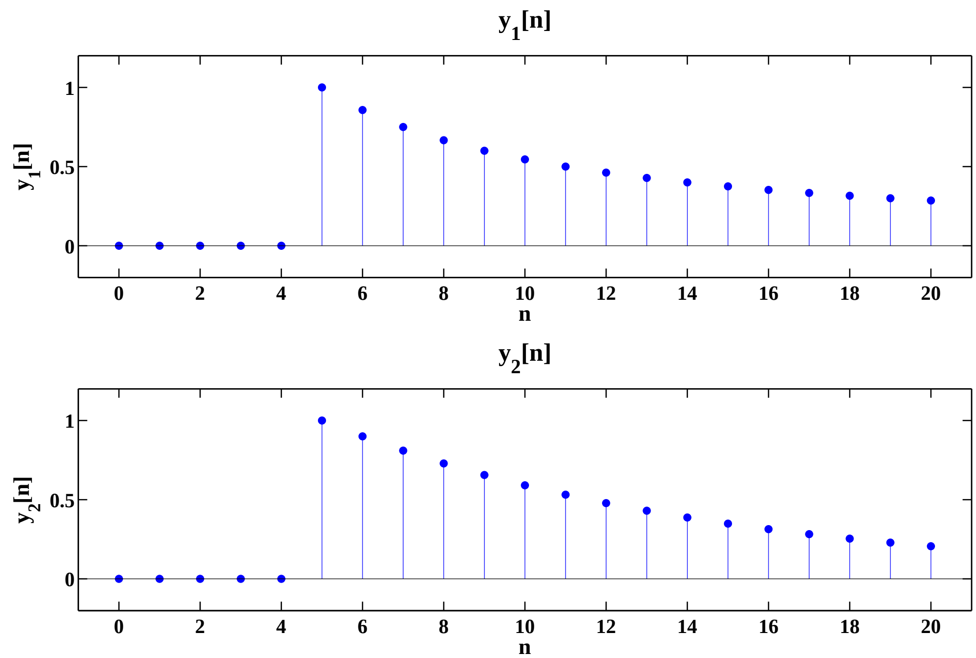

FIGURE 2.13: System responses with respect to input signal $x[n] = \delta[n-5]$.

```
xylimit = [n(1)-1,n(end)+1,min(yn1)-0.2,max(yn1)+0.2];
subplot(2,1,1)
stem(n,yn1,'fill'); axis(xylimit)
xlabel('n','fontsize',LFS); ylabel('y_1[n]','fontsize',LFS);
title('y_1[n]','fontsize',TFS);
subplot(2,1,2)
stem(n,yn2,'fill'); axis(xylimit)
xlabel('n','fontsize',LFS); ylabel('y_2[n]','fontsize',LFS);
title('y_2[n]','fontsize',TFS)
```

8. (a)

$$y[n] = \frac{1}{5}\left(x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]\right)$$

(b)

$$h[n] = u[n] - u[n-5]$$

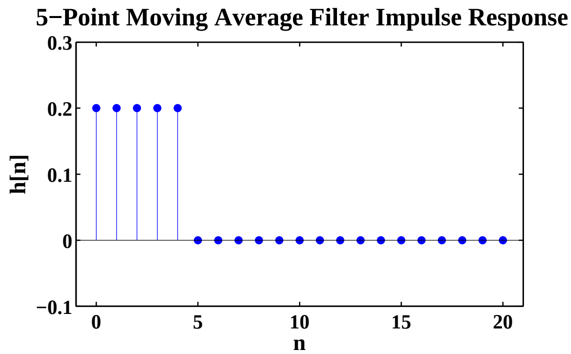

FIGURE 2.14: Impulse response of a 5-point moving average filter.

(c) Block diagram.

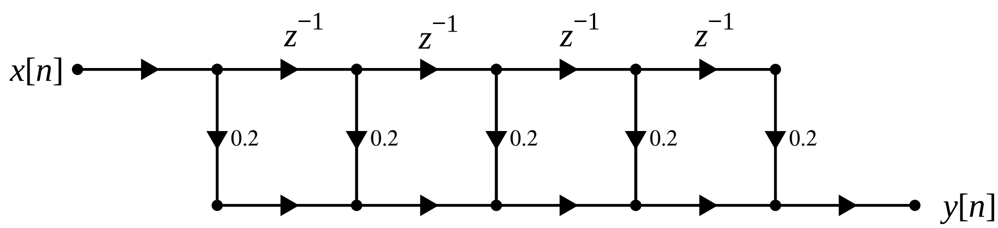

FIGURE 2.15: Block diagram of a 5-point moving average filter.

MATLAB script:

```
% P0208: Plot the 5-point moving average filter
%       y[n] = 1/5*(x[n]+x[n-1]+x[n-2]+x[n-3]+x[n-4]);
close all; clc
n = 0:20;
xn = delta(n(1),0,n(end))';
hn = filter(ones(1,5)/5,1,xn);
%Plot
hf = figconfg('P0208','small');
xylimit = [n(1)-1,n(end)+1,min(hn)-0.1,max(hn)+0.1];
stem(n,hn,'fill'); axis(xylimit)
```

```
xlabel('n','fontsize',LFS); ylabel('h[n]','fontsize',LFS);
title('5-Point Moving Average Filter Impulse Response',...
    'fontsize',TFS);
```

9. (a) Proof:

$$\sum_{n=0}^{\infty} a^n = 1 + a + a^2 + \cdots$$

$$a\sum_{n=0}^{\infty} a^n = a + a^2 + a^3 + \cdots$$

$$(1-a)\sum_{n=0}^{\infty} a^n = 1 + (a-a) + (a^2 - a^2) + \cdots + (a^\infty - a^\infty)$$

$$(1-a)\sum_{n=0}^{\infty} a^n = 1 + 0 + 0 + \cdots + 0$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

(b) Proof:

$$\sum_{n=0}^{N-1} a^n = \sum_{n=0}^{\infty} a^n - \sum_{n=N}^{\infty} a^n = \sum_{n=0}^{\infty} a^n - a^N \sum_{n=0}^{\infty} a^n$$

Substituting the result in part (a), we have

$$\sum_{n=0}^{N-1} a^n = (1-a^N)\sum_{n=0}^{\infty} a^n = \frac{1-a^N}{1-a}$$

10. (a) Solution:

$$x[-m] = \{-1, 2, 3, \underset{\uparrow}{1}\}$$

$$x[3-m] = \{\underset{\uparrow}{-1}, 2, 3, , 1\}$$

$$h[m] = \{\underset{\uparrow}{2}, 2(0.8)^1, 2(0.8)^2, 2(0.8)^3, 2(0.8)^4, 2(0.8)^5, 2(0.8)^6\}$$

$$x[3-m] * h[m] = \{\underset{\uparrow}{-2}, 4(0.8)^1, 6(0.8)^2, 2(0.8)^3\}$$

$$y[3] = \sum_{m=0}^{3} x[3-m] * h[m] = 6.064$$

(b) MATLAB script:

```
% P0210: Graphically illustrate the convolution sum
close all; clc
nx = 0:3;
x = [1,3,2,-1]; % input sequence
nh = 0:6;
h = 2*(0.8).^nh; % impulse response
nxf = fliplr(-nx); xf = fliplr(x); %folding
nxfs = nxf+3; % left shifting
[y1 y2 n] = timealign(xf,nxfs,h,nh);
y = y1.*y2;
y3 = sum(y);
%Plot
hf = figconfg('P0210');
subplot(5,1,1)
stem(nx,x,'fill')
axis([-4 7 min(x)-1 max(x)+1])
ylabel('x[k]','fontsize',LFS);
subplot(5,1,2)
stem(nh,h,'fill')
axis([-4 7 min(h)-1 max(h)+1])
ylabel('h[k]','fontsize',LFS);
subplot(5,1,3)
stem(nxf,xf,'fill')
axis([-4 7 min(x)-1 max(x)+1])
ylabel('x[-k]','fontsize',LFS);
subplot(5,1,4)
stem(nxfs,xf,'fill')
axis([-4 7 min(x)-1 max(x)+1])
ylabel('x[-k+3]','fontsize',LFS);
subplot(5,1,5)
stem(n,y,'fill')
axis([-4 7 min(y)-1 max(y)+1])
xlabel('k','fontsize',LFS);
ylabel('h[k]*x[-k+3]','fontsize',LFS);
```

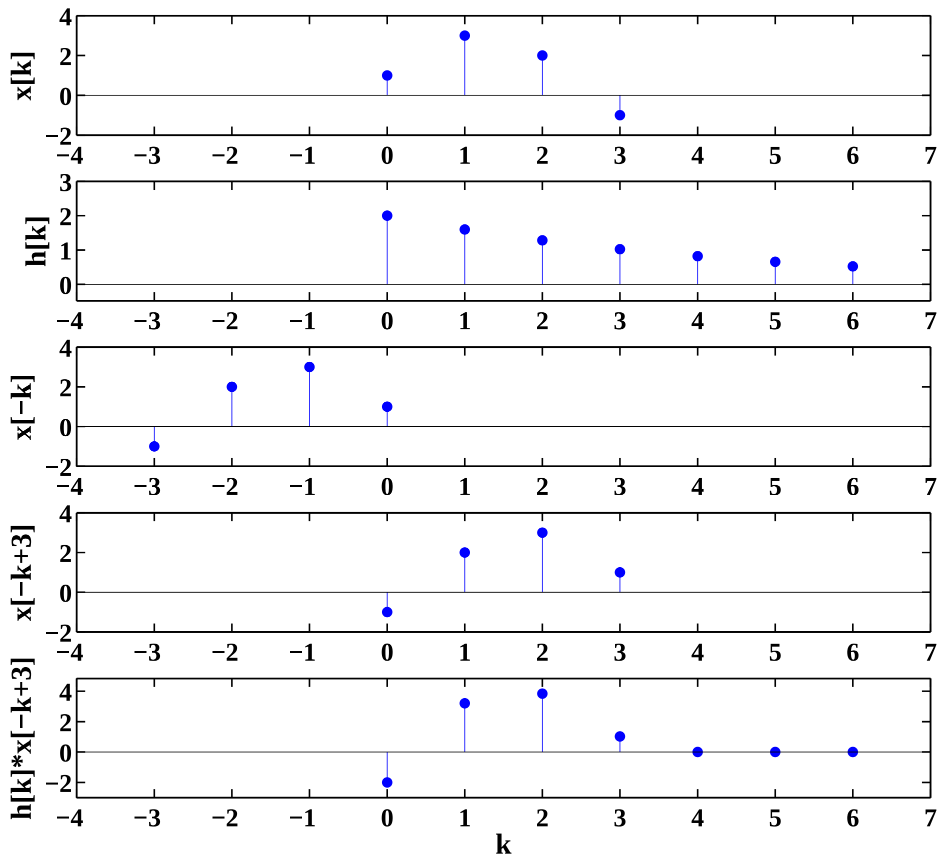

FIGURE 2.16: Graphically illustration of convolution as a superposition of scaled and scaled replicas.

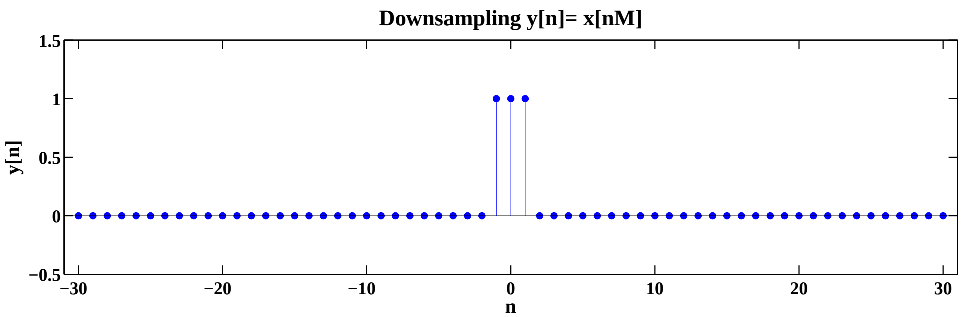

FIGURE 2.31: A down sampled signal $y[n]$ for $M = 20$.

```
% M = 5; % Part (b)
M = 20; % Part (c)
yind = mod(nx,M)==0;
y = x(yind);
ny = nx(yind)/M;
[x y n] = timealign(x,nx,y,ny);
hf = figconfg('P0222a','long');
stem(n,x,'fill')
axis([n(1)-1 n(end)+1 min(x)-0.5 max(x)+0.5])
xlabel('n','fontsize',LFS); ylabel('x[n]','fontsize',LFS);
title('x[n]','fontsize',TFS);
hf2 = figconfg('P0222b','long');
stem(n,y,'fill')
axis([n(1)-1 n(end)+1 min(y)-0.5 max(y)+0.5])
xlabel('n','fontsize',LFS); ylabel('y[n]','fontsize',LFS);
title('Downsampling y[n]= x[nM]','fontsize',TFS);
```

23. (a) $y[n] = x[-n]$ (Time-flip)
    linear, time-variant, noncausal, and stable

    (b) $y[n] = \log(|x[n]|)$ (Log-magnitude)
    nonlinear, time-invariant, causal, and unstable

    (c) $y[n] = x[n] - x[n-1]$ (First-difference)
    linear, time-invariant, causal, and stable

    (d) $y[n] = \text{round}\{x[n]\}$ (Quantizer)
    nonlinear, time-invariant, causal, and stable

32. MATLAB script:

```
% P0232: Use function 'filter' to study the impulse response and
%        step response of a system specified by LCCDE
close all; clc
N = 60;
n = 0:N-1;
b = [0.18 0.1 0.3 0.1 0.18];
a = [1 -1.15 1.5 -0.7 0.25];
[d nd] = delta(n(1),0,n(end));
[u nu] = unitstep(n(1),0,n(end));
y1 = filter(b,a,d);
y2 = filter(b,a,u);
% Plot:
hf = figconfg('P0232');
subplot(2,1,1)
stem(n,y1,'fill')
axis([n(1)-1,n(end)+1,min(y1)-0.2,max(y1)+0.2])
xlabel('n','fontsize',LFS)
title('Impulse Response','fontsize',TFS);
subplot(2,1,2)
stem(n,y2,'fill')
axis([n(1)-1,n(end)+1,min(y2)-0.5,max(y2)+0.5])
xlabel('n','fontsize',LFS)
title('Step Response','fontsize',TFS)
```

33. MATLAB script:

```
% P0233: Realize a first-order digital differentiator given by
%        y[n] = x[n] - x[n-1]
close all; clc
% Part (a):
n = -10:19;
x = 10*ones(1,length(n));

% % Part (b):
% nx1 = 0:9;
% x1 = nx1;
% nx2 = 10:19;
% x2 = 20-nx2;
```

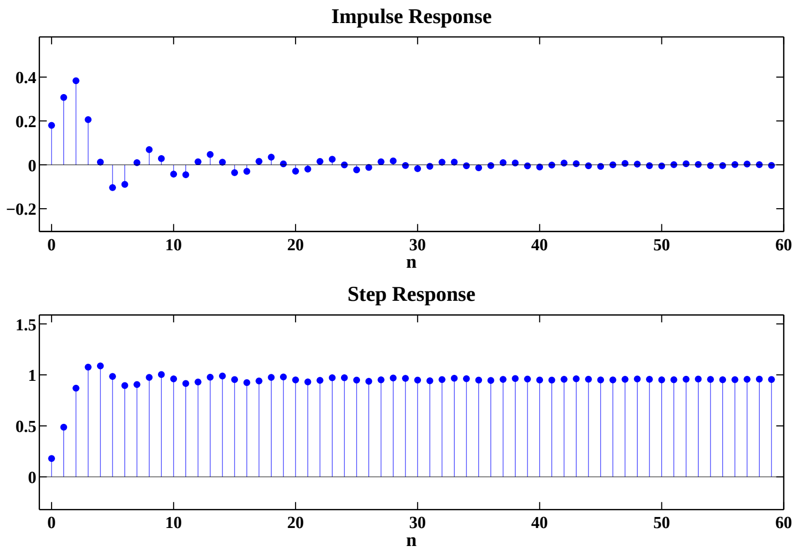

FIGURE 2.44: System impulse response and step response for first 60 samples using function `filter`.

```
% [x1 x2 n] = timealign(x1,nx1,x2,nx2);
% x = x1 + x2;

% % Part (c):
% n = 0:39;
% x = cos(0.2*pi*n-pi/2);

% Differentiator:
y = filter([1,-1],1,x);
% Plot:
hf = figconfg('P0233');
subplot(2,1,1)
stem(n,x,'fill')
axis([n(1)-1,n(end)+1,min(x)-1,max(x)+1])
xlabel('n','fontsize',LFS)
title('Input Signal x[n]','fontsize',TFS)
subplot(2,1,2)
stem(n,y,'fill')
```

```
axis([n(1)-1,n(end)+1,min(y)-1,max(y)+1])
xlabel('n','fontsize',LFS)
title('Response y[n] = x[n] - x[n-1]','fontsize',LFS)
```

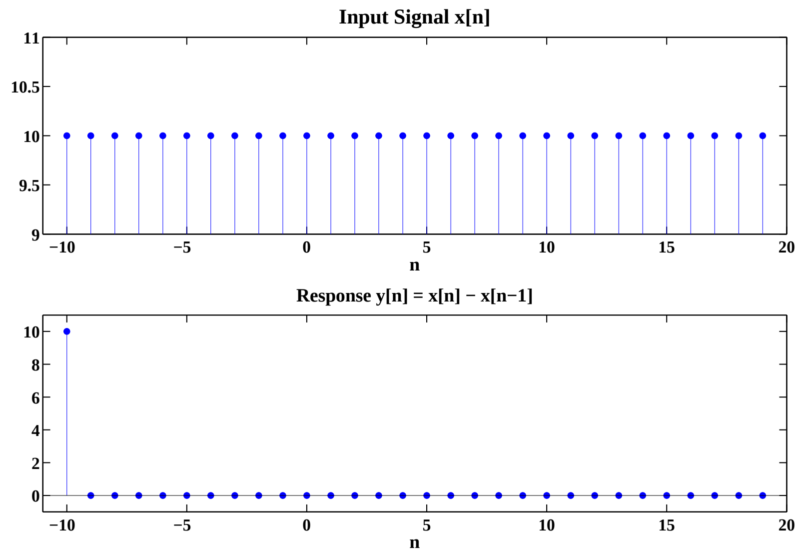

FIGURE 2.45: Differentiator output if input is $x[n] = 10\{u[n + 10] - u[n - 20]\}$.

34. MATLAB script:

```
% P0234: Use function 'filter' to study the impulse response
%        and step response of a system specified by LCCDE
close all; clc
N = 100;
n = 0:N-1;
b = 1;
a = [1 -0.9 0.81];
% a = [1 0.9 -0.81];
[d nd] = delta(n(1),0,n(end));
[u nu] = unitstep(n(1),0,n(end));
y1 = filter(b,a,d);
y2 = filter(b,a,u);
% Plot:
```

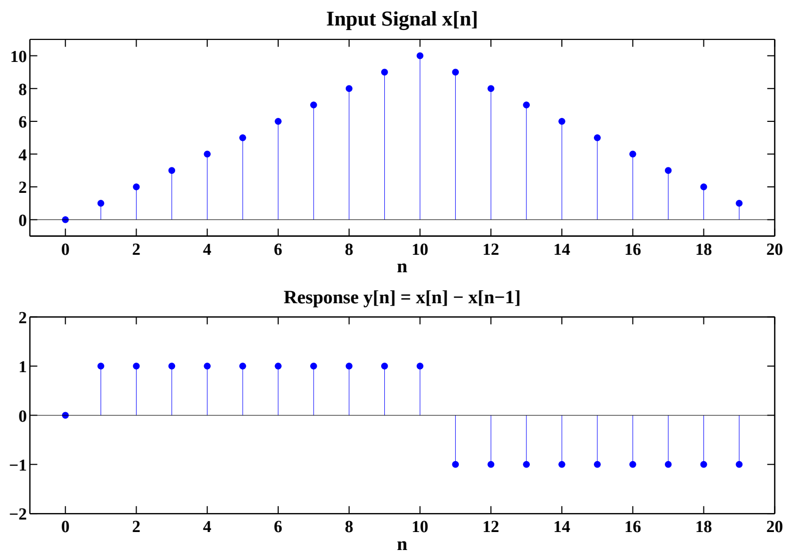

FIGURE 2.46: Differentiator output if input is $x[n] = n\{u[n] - u[n-10]\} + (20 - n)\{u[n-10] - u[n-20]\}$.

```
hf = figconfg('P0234a','long');
stem(n,y1,'fill')
axis([n(1)-1,n(end)+1,min(y1)-1,max(y1)+1])
xlabel('n','fontsize',LFS)
title('Impulse Response','fontsize',TFS)
hf2 = figconfg('P0234b','long');
stem(n,y2,'fill')
axis([n(1)-1,n(end)+1,min(y2)-1,max(y2)+1])
xlabel('n','fontsize',LFS)
title('Step Response','fontsize',TFS)
```

35. (a) $y(t) = x(t-1) + x(2-t)$
linear, time-variant, noncausal, and stable

(b) $y(t) = dx(t)/dt$
linear, time-invariant, causal, and unstable

(c) $y(t) = \int_{-\infty}^{3t} x(\tau)\mathrm{d}\tau$
linear, time-variant, noncausal, and unstable

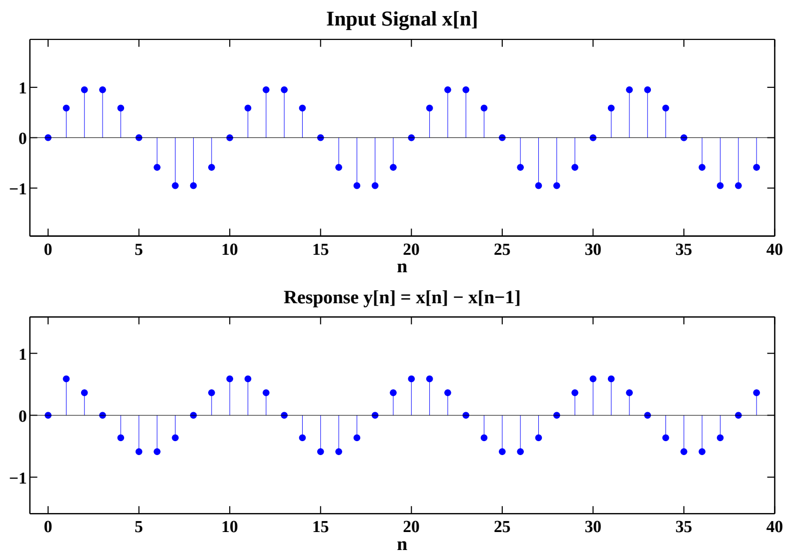

FIGURE 2.47: Differentiator output if input is $x[n] = \cos(0.2\pi n - \pi/2)\{u[n] - u[n-40]\}$.

(d) $y(t) = 2x(t) + 5$
nonlinear, time-invariant, causal, and stable

36. (a) Solution:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

if $t \in [-1, 1]$

$$y(t) = \int_{0}^{1+t} \tau/3 d\tau = \frac{(1+t)^2}{6}$$

if $t \in [1, 2]$

$$y(t) = \int_{-1+t}^{1+t} \tau/3 d\tau = \frac{2t}{3}$$

if $t \in [2, 4]$

$$y(t) = \int_{-1+t}^{3} \tau/3 d\tau = \frac{-t^2 + 2t + 8}{6}$$

$$y(t) = 0 \quad \text{otherwise}$$

```
x2 = 3*x2;
% Part (c):
[x3 nx3] = shift(x,nx,3);
[x3 nx3] = fold(x3,nx3);
% Plot:
hf = figconfg('P0239');
xlimit = [min([nx(1) nx1(1) nx2(1) nx3(1)])-1,...
    max([nx(end) nx1(end) nx2(end) nx3(end)])+1];
subplot(2,2,1)
stem(nx,x,'fill')
xlim(xlimit); ylim([min(x)-1,max(x)+1])
xlabel('n','fontsize',LFS); title('x[n]','fontsize',LFS)
subplot(2,2,2)
stem(nx1,x1,'fill')
xlim(xlimit); ylim([min(x1)-1,max(x1)+1])
xlabel('n','fontsize',LFS); title('2x[n-4]','fontsize',TFS)
subplot(2,2,3)
stem(nx2,x2,'fill')
xlim(xlimit)
ylim([min(x2)-1,max(x2)+1])
xlabel('n');
title('3x[n-5]')
subplot(2,2,4)
stem(nx3,x3,'fill')
xlim(xlimit); ylim([min(x3)-1,max(x3)+1])
xlabel('n','fontsize',LFS);
title('x[3-n]','fontsize',TFS)
```

40.

$$\begin{aligned} T\{a_1x_1[n] + a_2x_2[n]\} &= 10(a_1x_1[n] + a_2x_2[n])\cos(0.25\pi n + \theta) \\ &= a_1y_1[n] + a_2y_2[n] \end{aligned}$$

The system is linear.

$$\begin{aligned} T\{x[n-n_0]\} &= 10x[n-n_0]\cos(0.25\pi n + \theta) \\ &\neq y[n-n_0] = 10x[n-n_0]\cos(0.25\pi(n-n_0) + \theta) \end{aligned}$$

The system is time-variant.

$$h[n] = 10\delta[n]\cos(0.25\pi n + \theta)$$

The system is causal and stable.

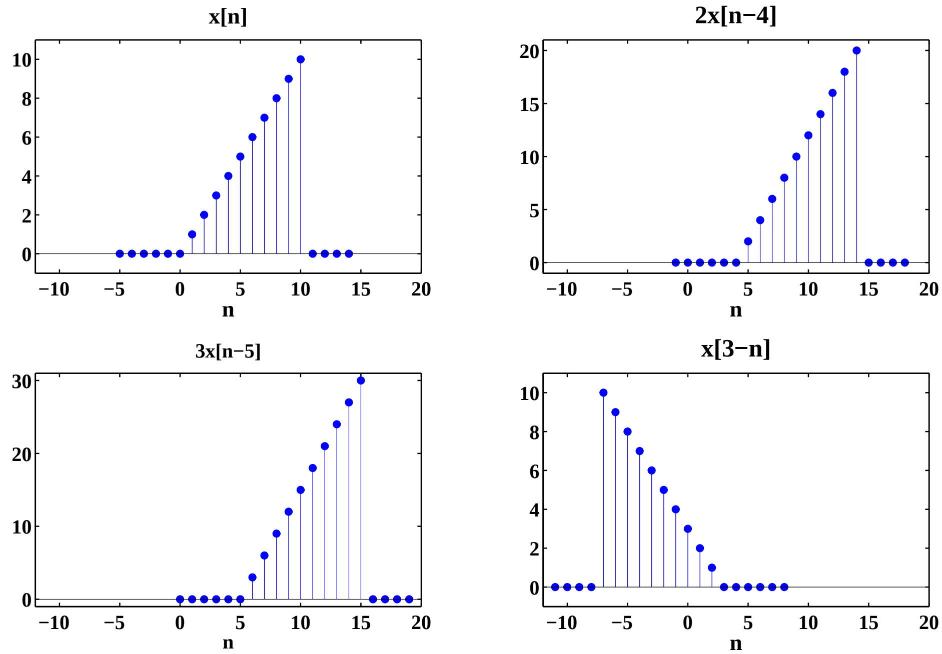

FIGURE 2.57: $x[n]$, $2x[n-4]$, $3x[n-5]$, and $x[3-n]$.

41. Comments: The system is unstable.

    MATLAB script:

```
% P0241: Compute and plot the output of the discrete-time system
%        y[n] = 5y[n-1]+x[n], y[-1]=0
close all; clc
n = 0:1e3;
x = ones(1,length(n));
y = filter(1,[1 -5],x);
% Plot:
hf = figconfg('P0241','small');
stem(n,y,'fill')
axis([n(1)-1 n(end)+1 min(y)-1 max(y)+1])
xlabel('n','fontsize',LFS);
title('y[n] = 5y[n-1]+x[n], y[-1]=0','fontsize',TFS)
```

42. MATLAB script:

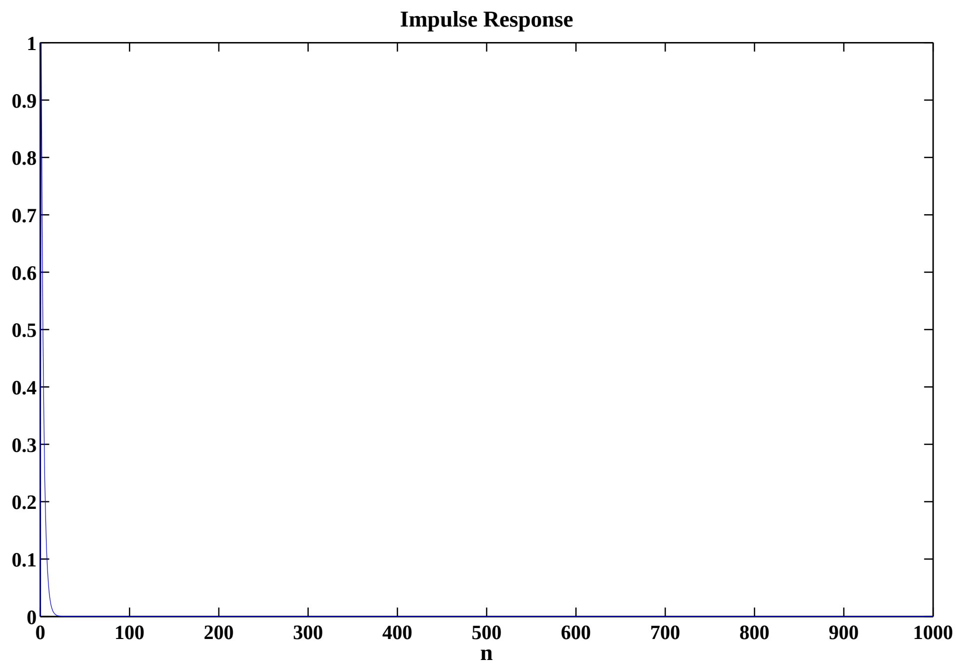

FIGURE 2.67: Impulse response for $a = 0.7$.

51. (a) Solution:

$$\begin{aligned}
h[n] * g[n] &= \left(\sum_{k=0}^{\infty} a_k \delta[n-kD]\right) * \left(\sum_{l=0}^{\infty} b_l \delta[n-lD]\right) \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} a_k b_l \delta[n-kD] * \delta[n-lD] \\
&= \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} a_k b_l \delta[n-kD-lD] \\
&= \sum_{k=0}^{\infty} c_k \delta[n-kD] = \delta[n]
\end{aligned}$$

Hence, we can conclude

$$\begin{cases} c_0 = a_0 b_0 = 1 \\ c_k = \sum_{m=0}^{k} a_{k-m} b_m = 0, \quad k > 0 \end{cases}$$

(b) Solution:

$$\begin{cases} c_0 = a_0 b_0 = 1 \\ c_1 = a_0 b_1 + a_1 b_0 = 0 \\ c_2 = a_0 b_2 + a_1 b_1 + a_2 b_0 = 0 \end{cases} \Longrightarrow \begin{cases} b_0 = a_0^{-1} \\ b_1 = -a_1 a_0^{-2} \\ b_2 = -a_2 a_0^{-2} + a_1^2 a_0^{-3} \end{cases}$$

(c) Solution:
Combined the conditions and the results of previous parts, we have

$$b_k + 0.5 b_{k-1} + 0.25 b_{k-2} = 0, \quad b_0 = 1,\ b_1 = -0.5,\ b_2 = 0$$

k	0	1	2	3	4	5	6	7	$\cdots$
b_k	1	-0.5	0	0.5^3	-0.5^4	0	0.5^6	-0.5^7	$\cdots$

We can conclude that

$$b_k = \begin{cases} 0.5^k, & k = 3l \\ -0.5^k, & k = 3l+1 \\ 0, & k = 3l+2 \end{cases} \quad l = 0, 1, 2, \ldots$$

52. (a) Solution:

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] = \sum_{k=-\infty}^{n} \delta[k]$$

$$s[n] = h[n] * u[n] = h[n] * \sum_{k=0}^{\infty} \delta[n-k] = \sum_{k=0}^{\infty} h[n] * \delta[n-k]$$
$$= \sum_{k=0}^{\infty} h[n-k]$$

if $\sum_{k=0}^{\infty} h[n-k] = 0$, for all $n < 0$

$$\left.\begin{array}{c} \sum_{k=0}^{\infty} h[-1-k] = \sum_{k=0}^{\infty} h[-2-k] + h[-1] = 0 \\ \sum_{k=0}^{\infty} h[-2-k] = 0 \end{array}\right\} \Longrightarrow h[-1] = 0$$

Follow the above procedure, we can prove by mathematical induction that $h[n] = 0$, for all $n < 0$. Hence, we conclude a system is causal if the step response $s[n]$ is zero for $n < 0$.

(b) Solution:

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[n-m] x[m]$$