Name:

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| --- | --- | --- |
| Problem | Points | Score |
| 1(a) | 10 |  |
| 1(b) | 10 |  |
| 1(c) | 5 |  |
| 1(d) | 5 |  |
| 1(e) | 5 |  |
| 1(f) | 5 |  |
| 2(a) | 15 |  |
| 2(b) | 15 |  |
| 3(a) | 10 |  |
| 3(b) | 10 |  |
| 3(c) | 10 |  |
| Total | 100 |  |

Notes:

1. The exam is closed books and notes except for one double-sided sheet of notes.
2. Please indicate clearly your answer to the problem.
3. If I can’t read or follow your solution, it is wrong and no partial credit will be awarded.

**Problem No. 1**: Consider the linear time-invariant constant coefficient difference equation:
$y\left[n\right]=0.5y\left[n-2\right]+0.25x[n-1]$. Assume the initial conditions are zero.

(10 pts) (a) Compute the z-Transform.

(10 pts) (b) Find the impulse response by taking the inverse z-Transform.

(5 pts) (c) Apply an impulse function to the difference equation and compute the impulse response.

(5 pts) (d) Demonstrate that the answers to (b) and (c) match.

(5 pts) (e) Apply a unit step function. Compute the output using z-Transforms. Demonstrate that the value you obtain for large values of time (e.g., n=10e06) makes sense.

(5 pts) (f) Is the system stable? Justify your answer using the results above.

**Problem No. 2**: Assuming a sample frequency of 8 kHz:

(15 pts) (a) Design a second-order digital resonator such that resonant frequency is 500 Hz and the 3 dB bandwidth is 100 Hz.

(15 pts) (b) Plot the location of the poles of this filter in the z-plane.

**Problem No. 3**: A linear system is described by the difference equation:

$$v\left[n\right]=x\left[n\right]+v\left[n-1\right]$$

$$y\left[n\right]=v\left[n\right]+v\left[n-2\right]+v[n-4]$$

(10 pts) (a) Determine the transfer function.

(10 pts) (b) Draw the direct form realization.

(10 pts) (c) Draw the canonical realization.