Name:

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| Problem | Points | Score |
| 1(a) | 15 |  |
| 1(b) | 15 |  |
| 2(a) | 15 |  |
| 2(b) | 15 |  |
| 2(c) | 5 |  |
| 3(a) | 15 |  |
| 3(b) | 10 |  |
| 43c) | 10 |  |
| Total | 100 |  |

Notes:

1. The exam is closed books and notes except for four double-sided sheet of notes.
2. Please indicate clearly your answer to the problem.
3. If I can’t read or follow your solution, it is wrong and no partial credit will be awarded.

**Problem No. 1**: Fast Fourier Transform

(15 pts) (1a) Direct multiplication of two complex numbers (a+jb)(c+jd) requires four real multiplications and two real additions. By properly arranging terms, show that it is possible to obtain the above multiplication using three real multiplications and five real additions.

$$\left(a+jb\right)\left(c+jd\right)=ac+jad+jbd-bd=\left(ac-bd\right)+j(bc+ad)$$

Three real multiplications and three additions

$$M\_{1}=a\*(d-c)$$

$$M\_{2}=b\*(c+d)$$

$$M\_{3}=c\*(a+b)$$

Two more additions

$$M\_{3}-M\_{2}=ac-bd$$

$$M\_{3}+M\_{1}=bc+ad$$

Finally the three real multiplications and five real additions put together

$$\left(M\_{3}-M\_{2}\right)+( M\_{3}+M\_{1})$$

(15 pts) (1b) Explain the key concepts behind the Fast Fourier Transform that allow a Discrete Fourier Transform to be computed faster and yet achieve the exact same result.

It takes more arithmetical operations to compute the DFT than computing the FFT. Discrete Fourier transform takes O (N2) arithmetical operations, while the FFT can compute the same Discrete Fourier transform in only O ( N log2 N) operations. FFT only do have of the computation to get the same results as DFT. Evaluating the DFT directly involve (N2) complex multiplication and complex additions N(N-1). FFT can compute the same result with only (N/2)log2(N) complex multiplications and complex addition Nlog2(N).

**Problem No. 2**: FIR Filter Design

The Hann window function can be written as:  where wR[n] is the rectangular window of length M+1.

(15 pts) (2a) Express the DTFT of w[n] in terms of the DTFT of wR[n].

$$w\left[n\right]=\left[0.5-0.5\*0.5\left(e^{j\frac{2πn}{M}}-e^{-j\frac{2πn}{M}}\right)\right]w\_{R}[n]$$

$$=0.5w\_{R}\left[n\right]-0.25w\_{R}\*\left[n\right]e^{j\frac{2πn}{M}}+0.25w\_{R}\left[n\right]\*e^{-j\frac{2πn}{M}}$$

$$w\_{R}[e^{jw}]=0.5w\_{R}\left[e^{jw}\right]-0.25w\_{R}\*\left[e^{jw}\right]e^{j\frac{2πn}{M}}+0.25w\_{R}\left[e^{jw}\right]\*e^{-j\frac{2πn}{M}}$$

$$w\_{R}[e^{jw}]=0.5w\_{R}\left[e^{jw}\right]-0.25w\_{R}(e^{j\left(w-\frac{2πn}{M}\right)})+0.25w\_{R}(e^{j\left(w+\frac{2πn}{M}\right)})$$

(15 pts) (2b) Explain why the Hann window has the wider mainlobe but lower sidelobes than the rectangular window of the same length.

The first term effects the mainlobe by making it wider.The second and third terms widen the mainlobe of Hann window and the sidelobes are lowed by the scaling factor.

(5 pts) (2c) Explain why the width of the main lobe is important. Give an example.

The width of the main lobe is important because it controllers the spectral resolution. A narrow main lobe results in a better resolution. A narrow main lobe will reproduce the original signal but that is shifted.

**Problem No. 3**: Let x[n] be an input signal and h[n] denote the causal and stable IIR system. First, x[n] is filtered through h[n] to obtain output y1[n]. Next, the flipped signal, x[-n], is filtered through h[n] to obtain y2[n]. Finally, the output is computed by summing y[n] = y1[n] and y2[n].

(15 pts) (3a) Let hs[n] be the impulse response of the overall system (e.g., x[n] is the input, y[n] is the output). Derive an expression for hs[n] in terms of h[n].

$$y\left[n\right]=x\left[n\right]\*h\left[n\right]+x\left[-n\right]\*h\left[n\right]=x\left[n\right]\*h\_{s}[n]$$

and

$x\left[-n\right]\*h\left[n\right]=\sum\_{m=-\infty }^{\infty }x\left[-m\right]h\left[n-m\right]=\sum\_{m=-\infty }^{\infty }x\left[m\right]h\left[n+m\right]=x\left[n\right]\*h[-n]$ (1)

Plug in (1) into y[n]

$$y\left[n\right]=x\left[n\right]\*h\left[n\right]+x\left[n\right]\*h\left[-n\right]=x\left[n\right](h\left[n\right]+h\left[-n\right])$$

We can conclude that

$$h\_{s}\left[n\right]=h\left[n\right]+h[-n]$$

(10 pts) (3b) Determine the frequency response of hs[n] and discuss any important properties you observe.

$$H\_{s}\left(e^{jω}\right)=H\left(e^{jω}\right)+H\left(e^{-jω}\right)=2Re[H\left(e^{jω}\right)]$$

(10 pts) (3c) Will these properties hold if h[n] is an IIR filter? Explain the significance of your findings.

The properties will not hold if h[n] is an IIR filter since the system is not causal anymore. We can make the system causal by adding more delays to make the properties hold.