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ECE 4522: Digital Signal Processing
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Exam No. 2

Problem No. 1: An LTI system is described by the difference equation:

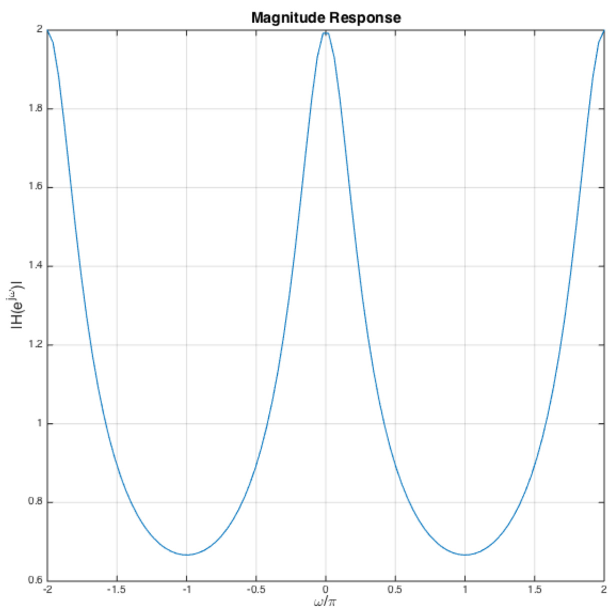
$$y[n] = 0.75y[n-1] - 0.125y[n-2] + x[n] - 0.25x[n-1]$$

(a) Derive an expression for the frequency response. Sketch your result - be as specific as possible.

$$Y(z) = 0.75z^{-1}Y(z) - 0.125z^{-2}Y(z) + X(z) - 0.25z^{-1}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 0.25z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$\boxed{H(e^{j\omega}) = \frac{1 - 0.25e^{-j\omega}}{1 - 0.75e^{-j\omega} + 0.125e^{j-2\omega}}}$$



(b) Derive an expression for the impulse response.

$$\begin{aligned} H(z) &= \frac{1 - 0.25z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}} \times \frac{z^2}{z^2} = \frac{z^2 - 0.25z}{z^2 - 0.75z + 0.125} \\ &= \frac{z(z-0.25)}{(z-0.5)(z-0.25)} = \frac{z}{z-0.5} \\ &= \frac{1}{1 - 0.5z^{-1}} \end{aligned}$$

$$\boxed{h[n] = \left(\frac{1}{2}\right)^n u[n]}$$

Problem No. 2: An 8-bit linear ADC has an input analog range of +/- 5V. The analog input signal is:

$$x_c(t) = 2cos(200\pi t) + 3sin(500\pi t)$$

The converter supplies data at a rate of 2048 bits/sec.
(a) What is the quantizer step size?
B = 8 bits

$$\Delta = \frac{Amplitude}{2^B} = \frac{10}{2^8} = 0.03906$$

(b) What is the SQNR in dB?

$$SQNR = 6.02B + 1.76 = 6.02(8) + 1.76$$
$$= 49.92 \quad dB$$

(c) If the quantizer was an audio signal of the same range, how could you improve the performance of the ADC?
To increase the performance of the 8 bit ADC, we can use the μ-law algorithm to reduce the quantization error, hence increasing the signal to quantization noise ratio (SQNR). A high SQNR is desired because that means that the signal is much greater than the noise, which will result in better audio quality.

Problem No. 3: Compute and sketch the N-point DFT in the range of $-(N-1) \leq n \leq (2N-1)$ for:
(a) $x[n] = \delta[n], N = 8$

$$W_8^{nk} = e^{-j\frac{2\pi nk}{8}}$$
$$X[k] = \sum_{n=-7}^{15} \delta[n] W_8^{nk} = W_8^{0k} = 1$$

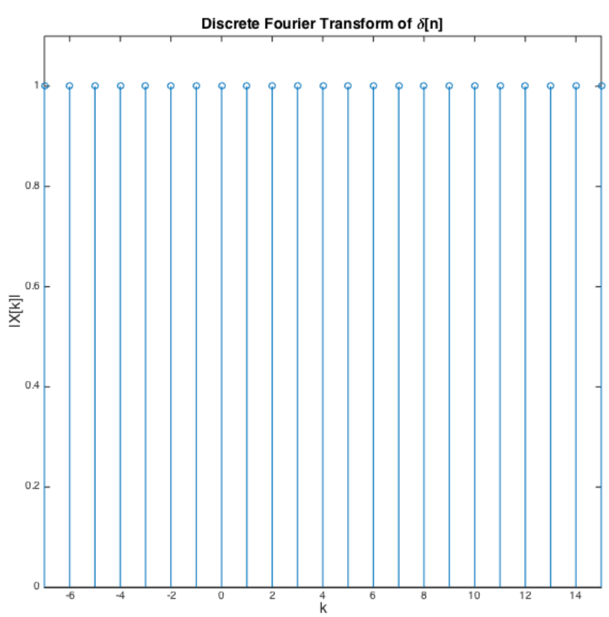


Figure 1: DFT of $\delta[n]$ in the range of -7 to 15.

(b) $x[n] = cos(6\pi n/15), N = 30$

$$X[k] = \sum_{n=-29}^{59} cos(\frac{6\pi}{15}n)W_{30}^{nk}$$
$$= \sum_{n=-29}^{59} \frac{1}{2}(e^{j\frac{2\pi 3n}{15}} + e^{-j\frac{2\pi 3n}{15}})W_{30}^{nk}$$

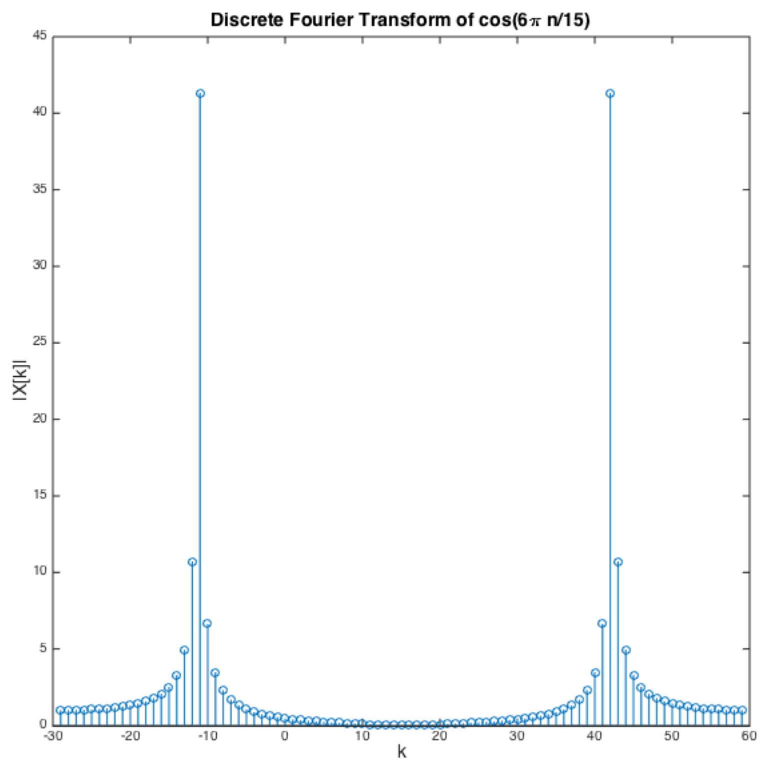


Figure 2: DFT of $cos(6\pi n/15)$ in the range of -29 to 59

(c) Explain the difference between the DFT of a sinewave when N is an integral number of periods, and when N is not an integral number of periods. Be a specific as possible and use sketches to demonstrate your points.

If the window captures an integral number of periods of the sine wave then the DFT of the sine wave is an impulse. In Figure 3a the window length is 1 period of the sine wave. The DFT is displayed in Figure 3b and there is an impulse at 2π because the frequency of the sine wave was 2π. If the window length is not an integral number of the period then the resulting DFT will have the picket fence effect. The picket fence effect is the additional energy that surrounds the impulse at 2π. Since the window is not an integral number of periods its need energy from other frequencies.

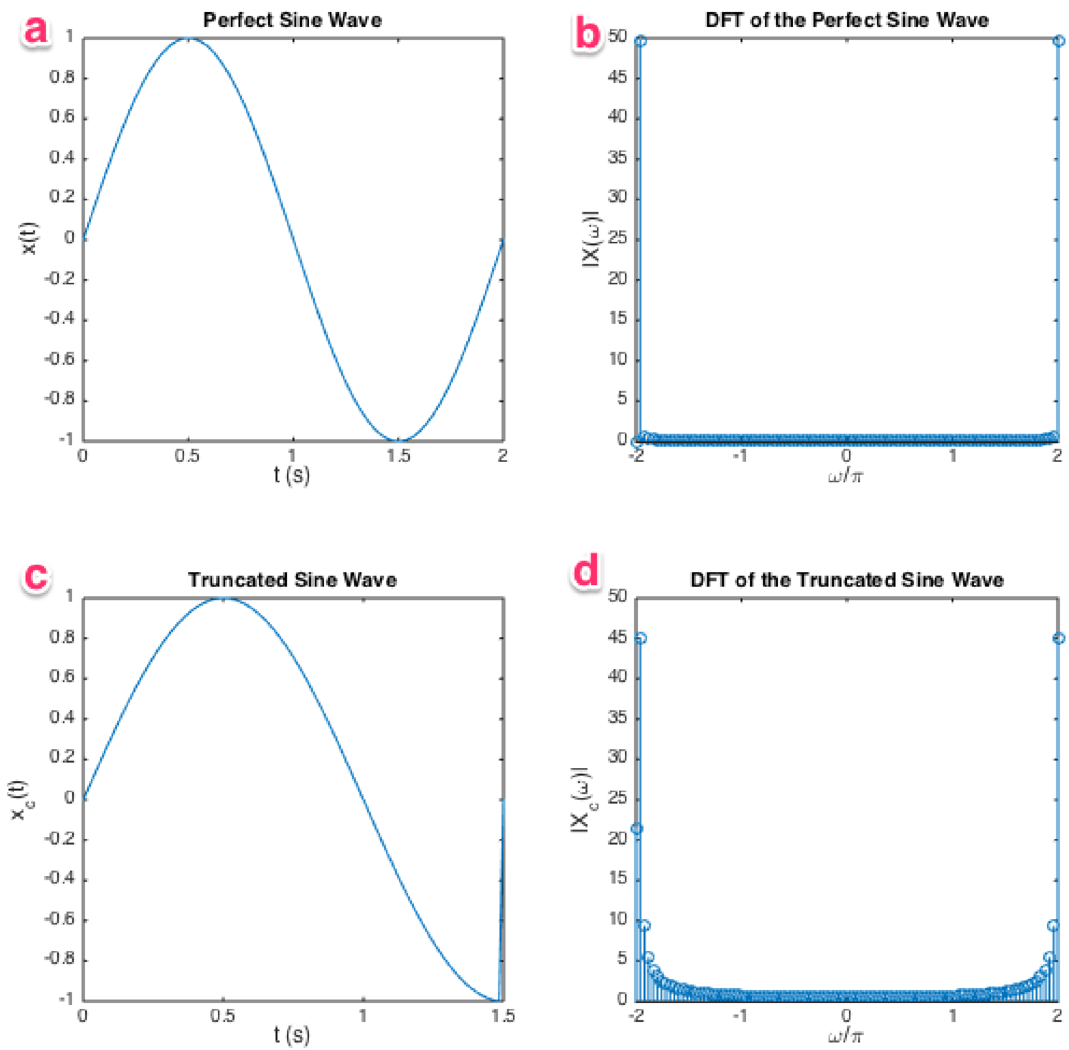


Figure 3: (a) Displays a sine wave with a period of 2π seconds. (b) Displays the Discrete Fourier Transform of the sine wave with an integer number of periods. (c) Displays the waveform of the truncated sine wave; the sine wave is cut off at $\frac{3}{2}\pi$ seconds. (d) Displays the Discrete Fourier Transform of the truncated sine wave.