ECE4522

Exam No.2

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Problem No. 1:

1.a. :

$$y[n] = 0.75y[n-1] - 0.125y[n-2] + x[n] - 0.25x[n-1] \xRightarrow{z-transform}$$

$$Y(e^{j\omega}) = 0.75e^{-j\omega}Y(e^{j\omega}) - 0.125e^{-2j\omega}Y(e^{j\omega}) + X(e^{j\omega}) - 0.25e^{-j\omega}X(e^{j\omega}) \Rightarrow$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1-0.25e^{-j\omega}}{1-0.75e^{-j\omega}+0.125e^{-2j\omega}} = \frac{1-0.25e^{-j\omega}}{(1-0.5e^{-j\omega})(1-0.25e^{-j\omega})} \quad (1)$$

The shape of the frequency response is determined by the impulse response or the coefficients of the difference equation. However, there is a strong dependence of the shape of the frequency response on the location of poles and zeros of the system. The zeros are the points where the frequency response dips down to touch the z-plane. The poles are points where the frequency response peaks. This system has one zero and two poles. One zero and one pole cancel each other and we can ignore them as the system is stable. In fact we can write:

$$H(e^{j\omega}) = \frac{1}{1-0.5e^{-j\omega}}$$

This system has a pole at z=0.5. So we see a peak at w=0. The frequency response, phase response and Group delay are illustrated in Fig 1.1.

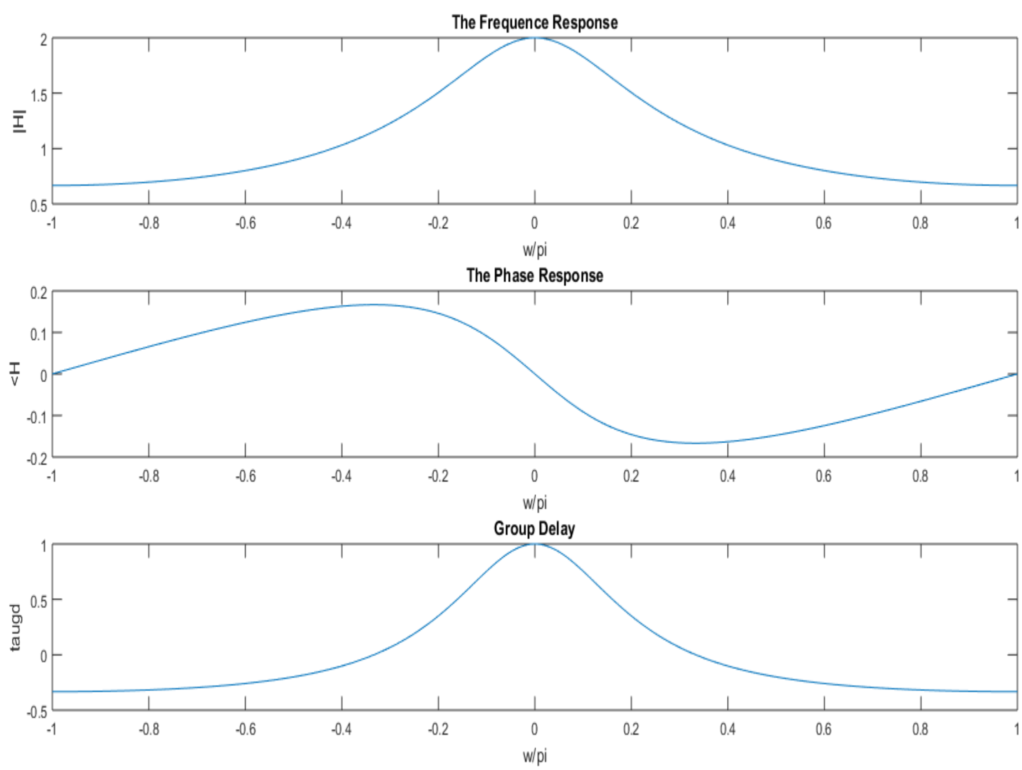

Fig 1.1

```
Matlab Code
% p1-a
clc
clear all
close all
om=linspace(-pi,pi,1000);
b=1; a=[1 -0.5];
H=freqz(b,a,om);
% grp delay is measured in samples
tau=grpdelay(b,a,om);
subplot(3,1,1), plot(om/pi,abs(H));
xlabel('w/pi')
ylabel('|H|')
title('The Frequence Response');
% angles are measured in units of pi rads
subplot(3,1,2), plot(om/pi,angle(H)/pi);
xlabel('w/pi')
ylabel('<H')
title('The Phase Response');
subplot(3,1,3), plot(om/pi,tau);
xlabel('w/pi')
ylabel('taugd')
title('Group Delay');
```

1.b. :

$$H(z) = \frac{1-0.25z^{-1}}{1-0.75z^{-1}+0.125z^{-2}} = \frac{1-0.25z^{-1}}{(1-0.25z^{-1})(1-0.5z^{-1})} = \frac{1}{1-0.5z^{-1}} \quad =>$$

$$h[n] = 0.5^n u[n]$$

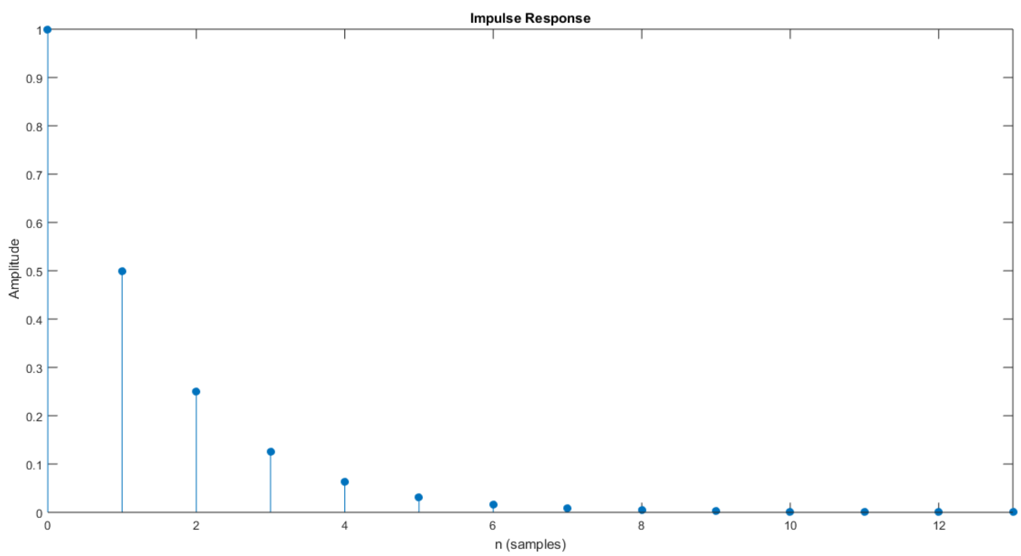

Fig 1.2

```
Matlab Code
%p1-b
clc
clear all
close all
b=1; a=[1 -0.5];
impz(b,a)
```

Problem No. 2:

2.a. :

The step size is:

$$\Delta = \frac{X_m}{2^B} = \frac{10}{256} = 0.039$$

2.b. :

SQNR(dB) = 10 log10 SQNR = 6.02B + 1.76 = 6.02 * 8 + 1.76 = 49.92

2.c. :

As the uniform quantizer is only optimal for uniformly distributed signal, it is not a good option for audio signals. The real audio signals (speech and music) are more concentrated near zeros and human ear is more sensitive to quantization errors at small values. So the solution is using non-uniform quantization such as µ-law quantization (in Europe, a similar standard, known as A-law quantization is used). In this technique, the quantization interval is smaller near zero. The µ-law quantization is illustrated in Fig 2.1.

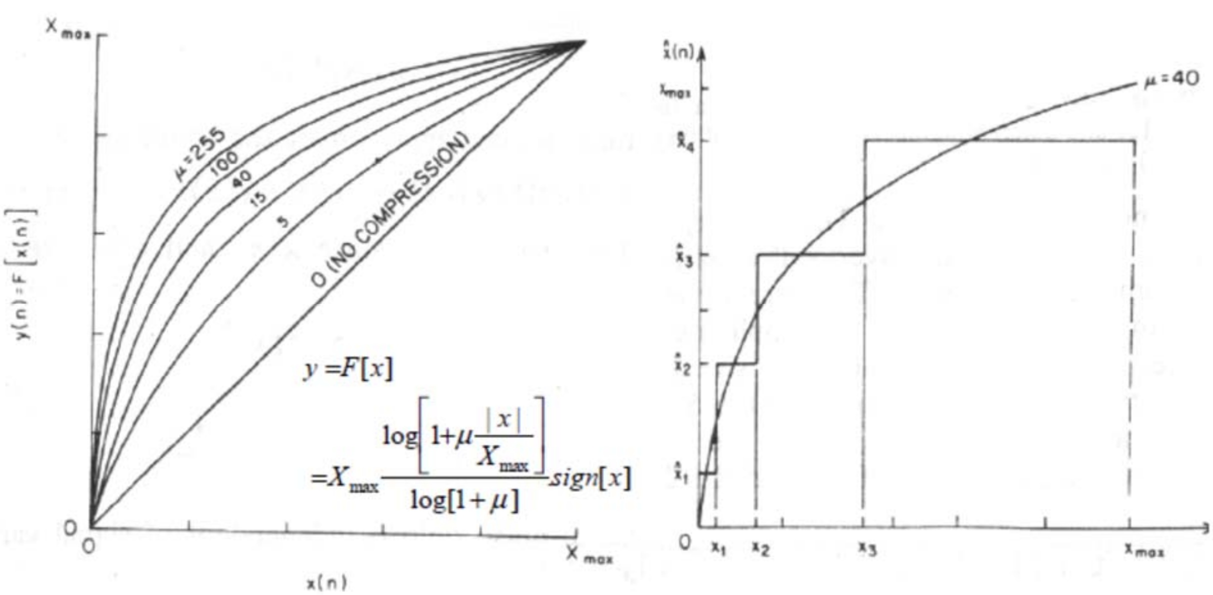

Fig 2.1

Problem No. 3:

The N-point Discrete Fourier Transform (DFT) X[k], 0 ≤ k ≤ N − 1 is defined by

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

3.a. :

$x[n] = \delta[n], N = 8$ => x = [1 0 0 0 0 0 0 0]

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{7} x[n]e^{-j\frac{2\pi}{N}kn} =>$$

X = [1 1 1 1 1 1 1 1]

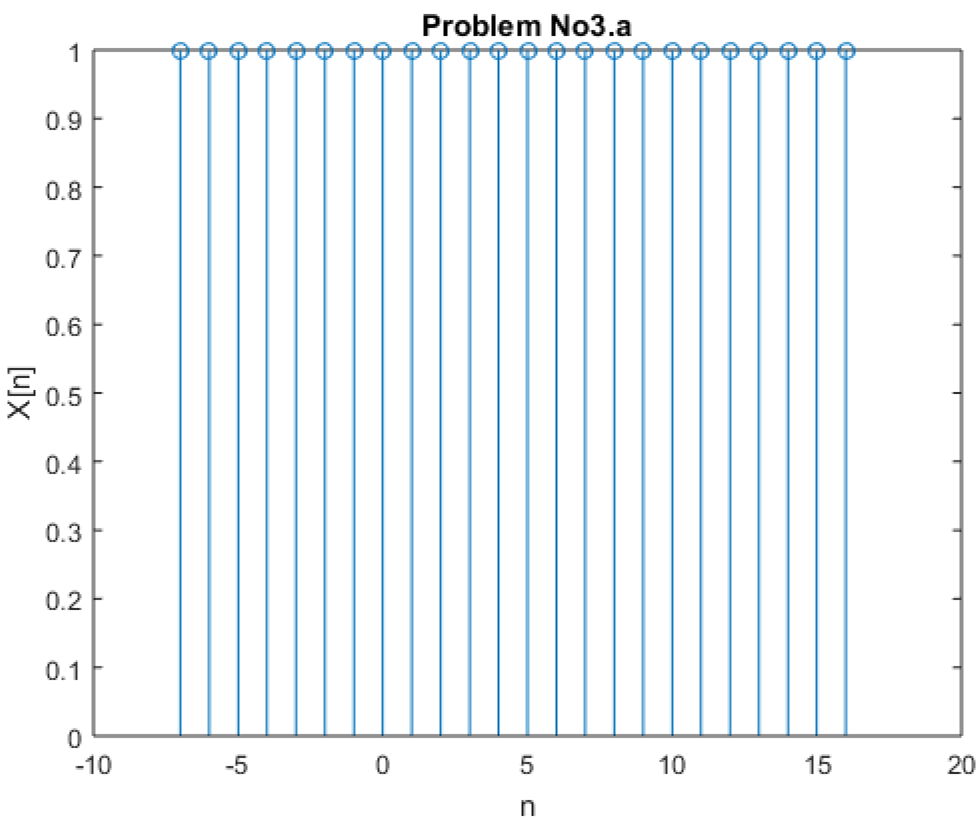

Fig 3.1

```
%p3-a: we can use also fft function.
clc
clear all
close all
x = [1 0 0 0 0 0 0 0]';
W = dftmtx(8);
X = W*x;
y=[X; X; X]';
n=[-7:1:16];
```

```
stem(n,y)
title('Problem No3.a')
xlabel('n')
ylabel('X[n]')
```

3.b. :

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

$$x[n] = \cos\left(\frac{6\pi n}{15}\right) = \frac{e^{j\frac{6\pi n}{15}} + e^{-j\frac{6\pi n}{15}}}{2} =>$$

$$X[k] = \sum_{n=0}^{29} \left(\frac{e^{j\frac{6\pi n}{15}} + e^{-j\frac{6\pi n}{15}}}{2}\right)(e^{-j\frac{2\pi}{N}kn}) = \frac{30}{2}[\delta(k-6) + \delta(k-24)]$$

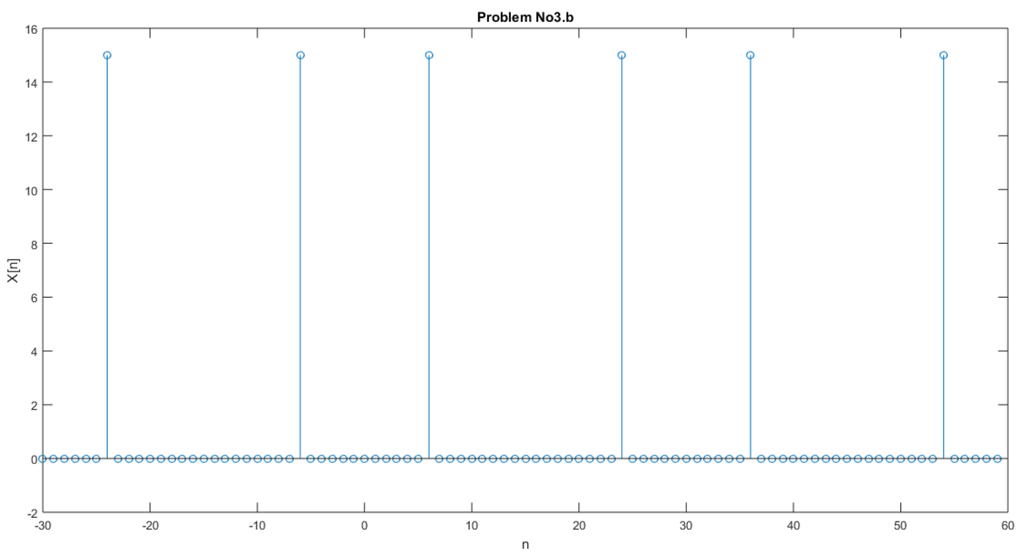

Fig 3.2

```
%p3-b
clc
clear all
close all
n=[0:1:29];
x = cos(12*pi*n/30);
W = dftmtx(30);
X = W*x';
y=[X; X; X]';
```

```
k=[-30:1:59];
stem(k,y)
title('Problem No3.b')
xlabel('n')
ylabel('X[n]')
```

3.c. :

An important application of the DFT is to numerically determine the spectral content of signals. When N is an integer number of period, we can find the exact spectral content of the signal. This situation is illustrated in Fig 3.3 and Fig 3.4.

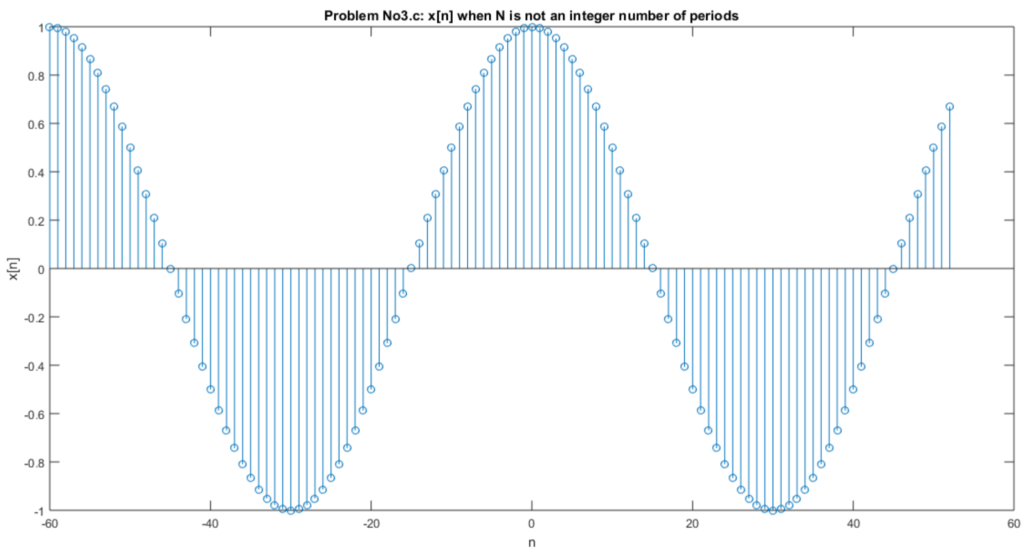

Fig 3.3

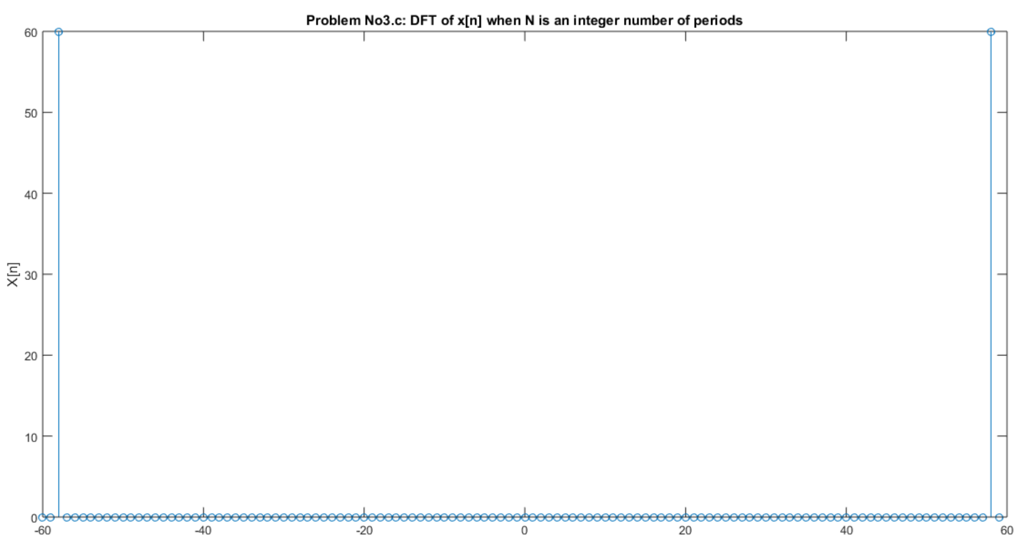

Fig 3.4

However, usually we do not know the spectral content of the signals; in this situation, when N is not an integer number of periods, DFT will be implemented on a truncated sine wave such as Fig 3.5. The DFT of this truncated sine wave is illustrated in Fig 3.6 and we can see two differences with Fig 3.4:

1. Leakage: The truncation of the signal causes the presence of harmonics which is called leakage.

2. Picket-fence Effect: The spectral peak is midway between sample locations. So there will be a drifting in frequency measurement. In general, the peaks in an FFT spectrum will be measured too low in level, and the valleys too high. This effect is reduced by increasing N, the number of DFT points.

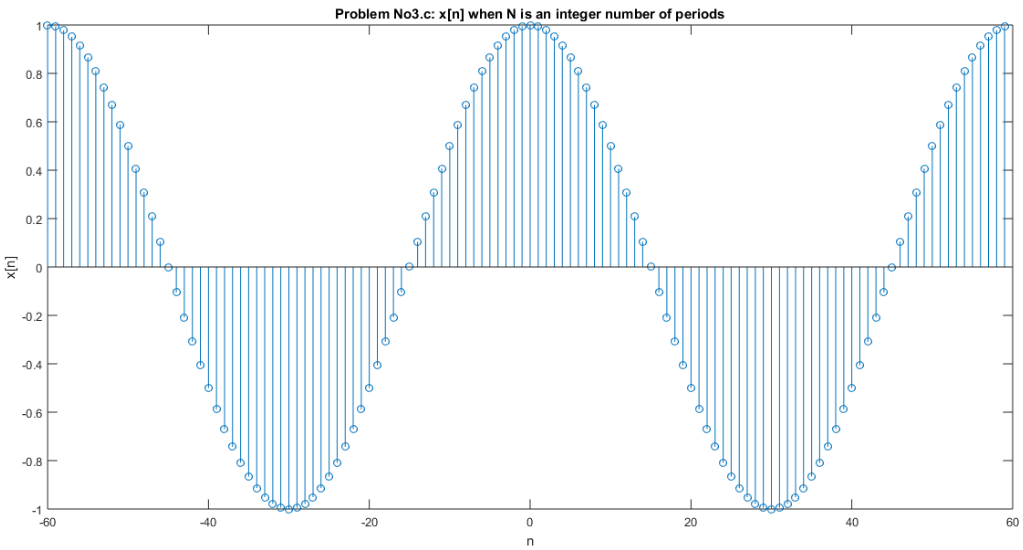

Fig 3.5

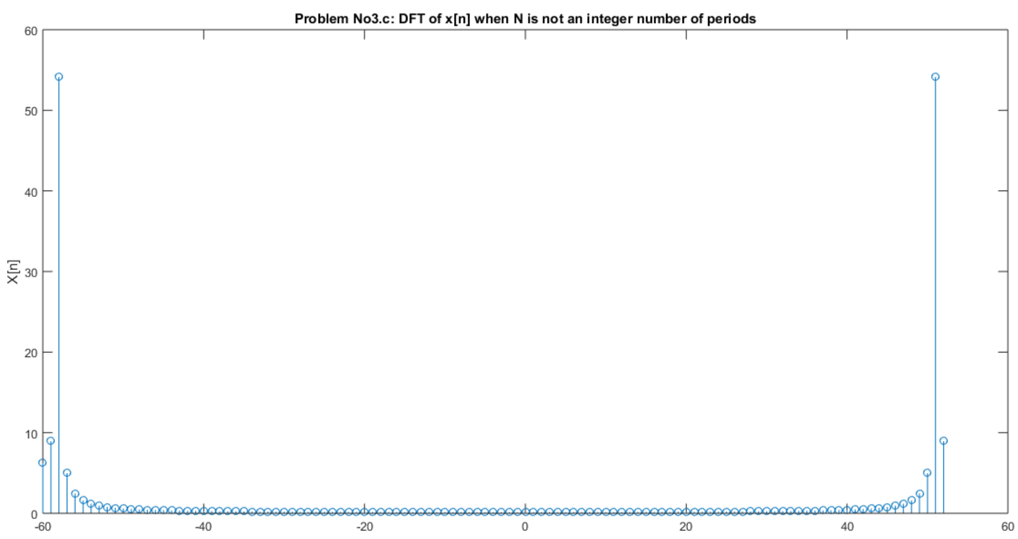

Fig 3.6

```
%p3-c
clc
clear all
close all
n0=[-60:1:59];
x0 = cos(pi*n0/30);
figure
stem(n0,x0)
title('Problem No3.c: x[n] when N is an integer number of periods')
xlabel('n')
ylabel('x[n]')


W0 = dftmtx(120);
X0 = W0*x0';
figure
stem(n0,abs(X0))
title('Problem No3.c: DFT of x[n] when N is an integer number of periods')
xlabel('n')
ylabel('X[n]')


x1=x0(1:113)
n1=n0(1:113)

figure
stem(n1,x1)
title('Problem No3.c: x[n] when N is not an integer number of periods')
xlabel('n')
ylabel('x[n]')

W0 = dftmtx(113);
X0 = W0*x1';
figure
stem(n1,abs(X0))
title('Problem No3.c: DFT of x[n] when N is not an integer number of
periods')
xlabel('n')
ylabel('X[n]')
```