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Exam 1 Redo
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Problem 1

Consider the LTCC difference equation: $y[n] = a_1 y[n-1] + a_2 y[n-2] + x[n] + b_1 x[n-1]$, where all coefficients are real numbers.

a) Derive the conditions under which the system is stable.

$$y(z) = a_1 y(z) z^{-1} + a_2 y(z) z^{-2} + x(z) + b_1 x(z) z^{-1}$$
$$y(z) - a_1 y(z) z^{-1} - a_2 y(z)^{-2} = x(z) + b_1 x(z) z^{-1}$$
$$y(z)(1 - a_1 z^{-1} - a_2 z^{-2}) = x(z)(1 + b_1 z^{-1})$$
$$\frac{y(z)}{x(z)} = \frac{1 + b_1 z^{-1}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$
$$H(z) = \frac{1 + b_1 z^{-1}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

The zero effectively has no dispute in the stability of the following system. Rather the decision relies much on a_1 and a_2. Since a_1 and a_2 are the poles of the system, for the system to maintain stability the magnitude of a_2 would have to be less than 1.

b) Assume $b_1 = 0$. Derive an expression for the impulse response and explain the role a_1 and a_2 play in the shape of this impulse response. Be as specific as possible. For example, if the signal oscillates, write an equation for the frequency of oscillation in terms of the coefficients.

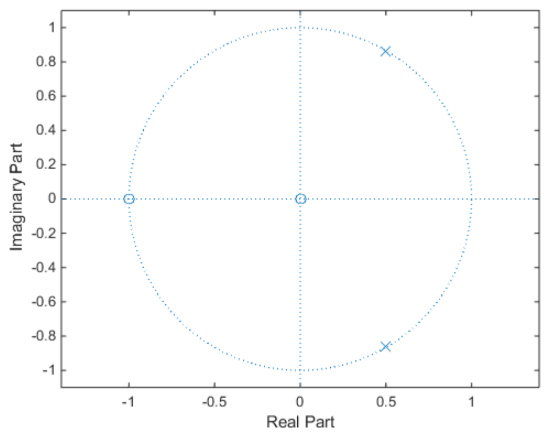
$$H(z) = \frac{1}{1 - a_1 z^{-1} - a_2 z^{-2}}$$
$$\frac{y(z)}{x(z)} = \frac{1}{1 - a_1 z^{-1} - a_2 z^{-2}}$$
$$y(z) = \frac{x(z)}{1 - a_1 z^{-1} - a_2 z^{-2}}$$
$$y(z)(1 - a_1 z^{-1} - a_2 z^{-2}) = x(z)$$
$$y(z) - a_1 y(z) z^{-1} - a_2 y(z)^{-2} = x(z)$$
$$y(z) = a_1 y(z) z^{-1} + a_2 y(z) z^{-2} + x(z)$$
$$y[n] = a_1 y[n-1] + a_2 y[n-2] + x[n]$$

Knowing that $b_1 = 0$ the transfer function takes the form above. The impulse was also derived from the original transfer function. The modified transfer function should be evaluated at $z = e^{j\omega}$ to develop an equation to describe the oscillations.

$$H(j\omega) = \frac{1}{1 - a_1 e^{-j\omega} - a_2 e^{-2j\omega}}$$

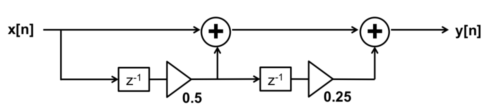
Taking the frequency response it is clear that the function would form a damped sinusoidal, depending on the severity of the damping on a_1 and a_2.
c)Assume the system is stable and $b_1 \neq 0$. Plot the location of the poles and zeroes in the z-plane and explain how these influence the frequency response. (Hint: describe the relationship between the z-transform and the frequency response.)

To convert a z transform to the frequency response, the z terms given in the transform are replaced by $e^{-j\omega}$. As shown in the unit circle below, while the poles rotate around the unit circle counterclockwise, ω is increasing up to π or $z = -1$. From this it is the negative half of the circle eventually ending up at $z = 1$ or $\omega = 0$. In the picture below the poles are located approximately at $z = e^{j\omega}$ and $z^* = e^{-j\omega}$. This means that the poles on the unit circle would give the frequency response peaks while the zeros would give dips in frequency.



Problem 2

Given the system shown:



a)Find the impulse response, h[n].
Using block diagram analysis the impulse response can be found.

$$h[n] = 1 + \frac{1}{2}x[n-1] + \frac{1}{8}x[n-2]$$

b) Find the transfer function, H[z].

$$
\begin{aligned}
y[n] &= x[n] + \frac{1}{2}x[n-1] + \frac{1}{8}x[n-2] \\
y[z] &= x[z] + \frac{1}{2}x[z]z^{-1} + \frac{1}{8}x[z]z^{-2} \\
y[z] &= x[z](1 + \frac{1}{2}z^{-1} + \frac{1}{8}z^{-2}) \\
\frac{y[z]}{x[z]} &= 1 + \frac{1}{2}z^{-1} + \frac{1}{8}z^{-2} \\
H[z] &= 1 + \frac{1}{2}z^{-1} + \frac{1}{8}z^{-2}
\end{aligned}
$$

Problem 3

Given the signal x[n] = [0 1 0 2] represents a continuous-time signal sampled at 2 Hz.
a) Find the frequency response, H(f), and sketch the magnitude as a function of frequency in Hz.

Since the signal given is an aperiodic signal, comprised of two impulses, the frequency response will be a continuous signal. This signal will mimic somewhat the shape of a cosine. First, rewrite the equation given into a z transform

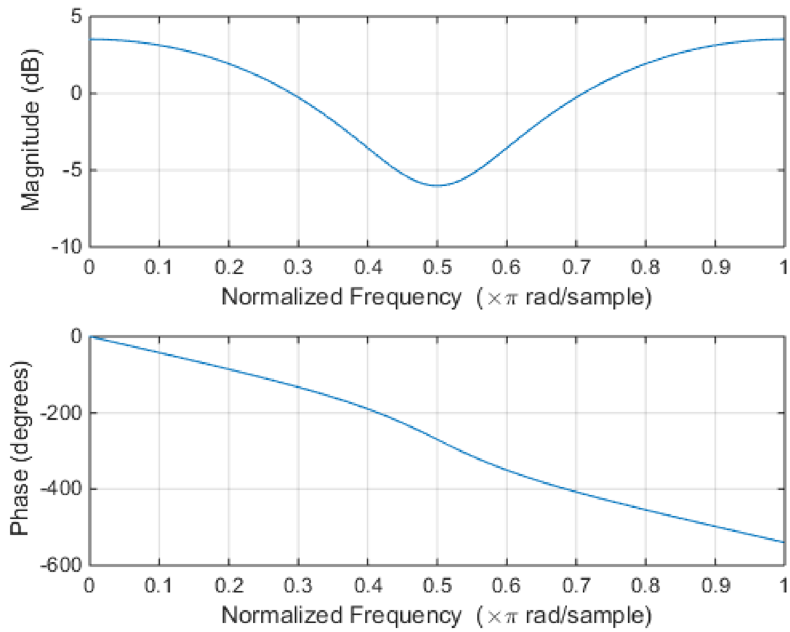
$$
\begin{aligned}
y[n] &= x[n-1] + 2x[n-3] \\
y[z] &= x[z]z^{-1} + 2x[z]z^{-3} \\
y[z] &= x[z](z^{-1} + 2z^{-3}) \\
H[z] &= z^{-1} + 2z^{-3}
\end{aligned}
$$

From this point we must convert to the frequency response by substituting $e^{j\omega}$ for each z. In this case $\omega = \frac{2\pi f}{F_s}$ and $F_s = 2Hz$.

$$H(e^{j\omega}) = e^{-j\omega} + 2e^{-3j\omega}$$

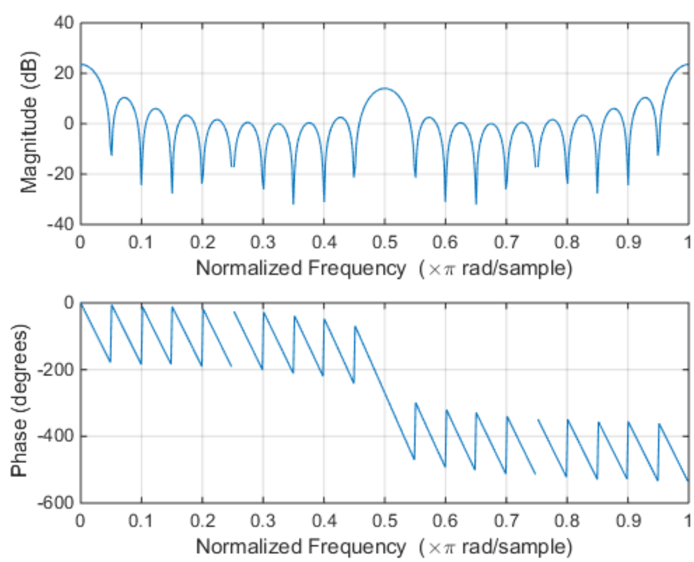
However, to convert to the frequency domain and be able to sketch the magnitude as a function of frequency, a Fourier Transform must be done.

$$X[k] = \sum x[n]e^{-j\omega n}$$



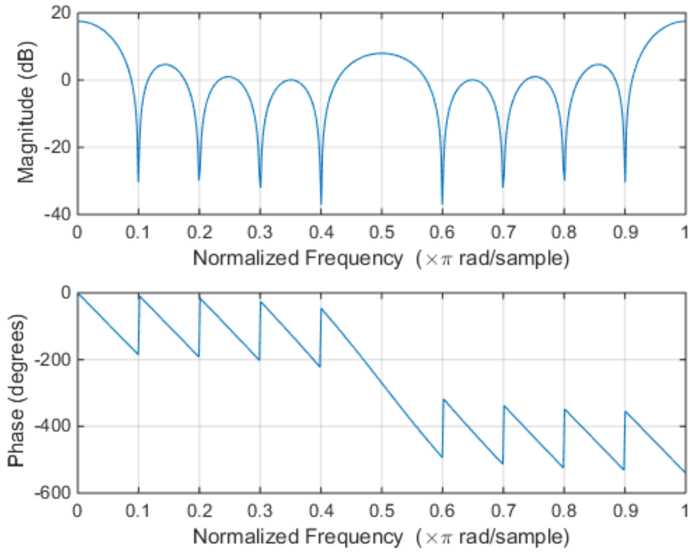
b) Next, assume the signal is periodic with a period of 10 samples. Sketch the frequency response and explain the relationship between this answer and the answer to a).

If the signal is then periodic with a period of 10 samples it will then resemble a collection of impulses, or a line spectrum that will form a signal discretely. The following sketch of the signal is below. Though they are line spectrum the Matlab command sketches the signal as a continuous one, through the command $freqz$.



c) How would your answer to (b) change if the period was 5 samples rather than 10 samples?

If the period was 5 samples rather than 10 samples, the sharpness of the Discrete Fourier Transform would not be as defined as the one with 10 samples as there was more line spectra to pronounce the curves. As a bonus the graph for 5 samples is found below.



Problem 4

The impulse response to a linear time-invariant system is given by: a) Draw a block diagram realization of the system (e.g, use delay lines, adders, and amplifiers).

Using all of the components to build block diagrams the following model has been constructed.

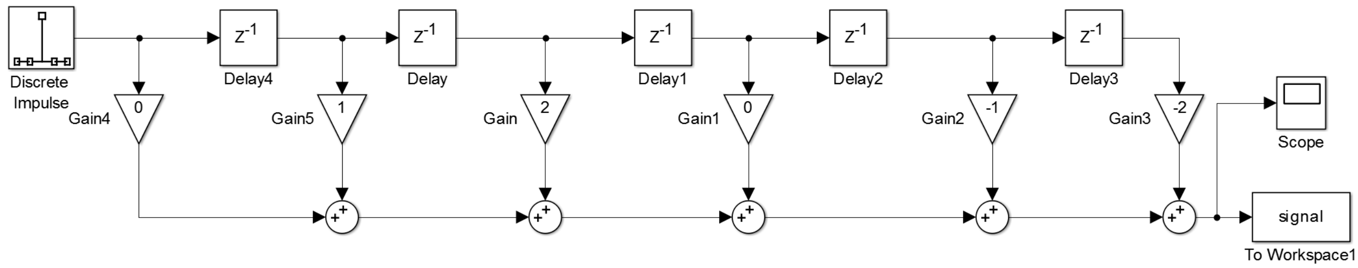


Figure 1: Block Diagram for the System

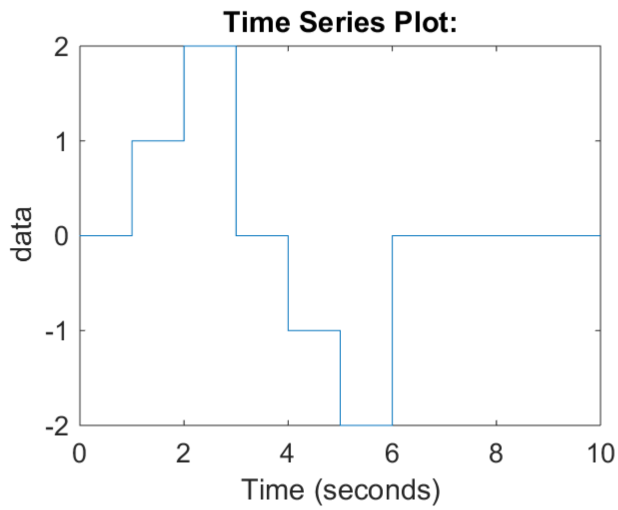
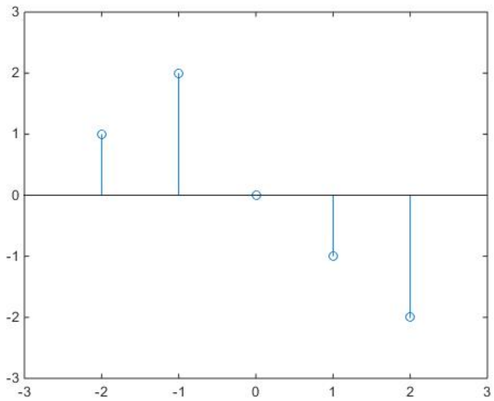
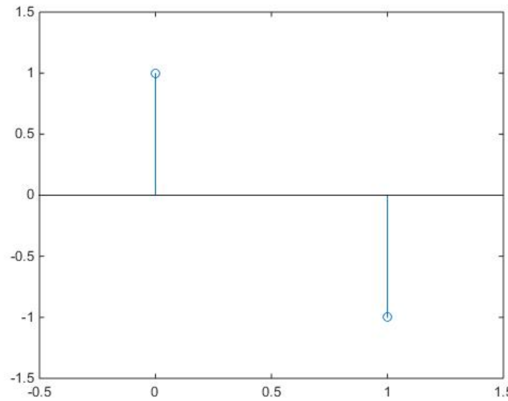


Figure 2: Response of the System

b) Suppose the input to this system is x[n] = [1 -1]. Compute the output. First, using Matlab plot both stem graphs.



(a) Stem Plot of h[n]



(b) Stem Plot of x[n]

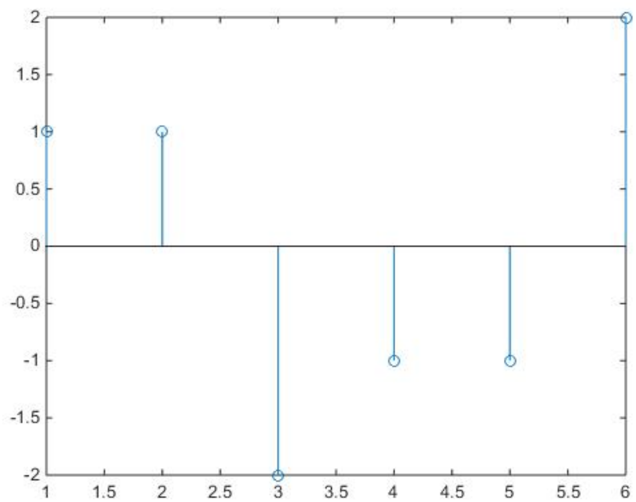


Figure 3: Two Signals Convolved (h[n]*x[n])

n	1	2	0	-1	-2	y[n]
0	1					1
1	-1	1				1
2		-1	1			-2
3			-1	1		-1
4				-1	1	-1
5					-1	2

Table 1: Graphical Method to Convolve Signals