

$$y(n) = 0.5y(n-1) + x(n)$$

$$x(n) = \delta(n)$$

n	x(n)	y(n-1)	y(n)
0	1	0	1
1	0	1	.5
2	0	.5	.25
3	0	.25	.125
4	0	0	;

$$h(n) \Rightarrow H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

Z-transform of the transfer function

Z-transforms

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Ex: $x(n) = \delta(n)$

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} \delta(n) z^{-n} \\ &= 1 \quad (n=0) \end{aligned}$$

Properties:

Linearity:

$$a_1 x_1(n) + a_2 x_2(n) \Leftrightarrow a_1 X_1(z) + a_2 X_2(z)$$

Delay:

$$x(n) \Leftrightarrow X(z)$$

$$x(n-D) \Leftrightarrow z^{-D} X(z)$$

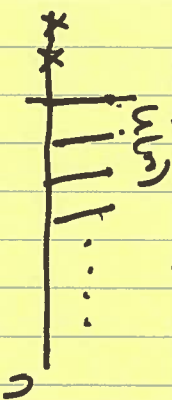
Convolution:

$$y(n) = h(n) * x(n)$$

$$Y(z) = H(z) X(z)$$

Example:

$$x(n) = u(n) - u(n-1)$$



$$x(n) = u(n)$$

$$X(z) = \sum_{n=-\infty}^{\infty} u(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (1) z^{-n}$$

 ~~$x(n) = u(n)$~~

$$x(n) = u(n-1)$$

$$X(z) = \sum_{n=0}^{\infty} z^{-n}$$

$$Z[u(n) - u(n-1)] = \sum_{n=0}^{\infty} z^{-n} + z^{-1} \sum_{n=0}^{\infty} z^{-n} = 1$$

$$\underbrace{X(z)}_{n=0} - z^{-1} X(z) = 1$$

$$X(z) = \frac{1}{1-z^{-1}} \Rightarrow u(n)$$

Region of convergence

$$ROC \triangleq \sum_{n=-\infty}^{\infty} x(n)z^{-n} \neq \infty$$

$$x(n) = [1, a, a^2, \dots, a^n] \quad x(n) = 0 \quad n < 0$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

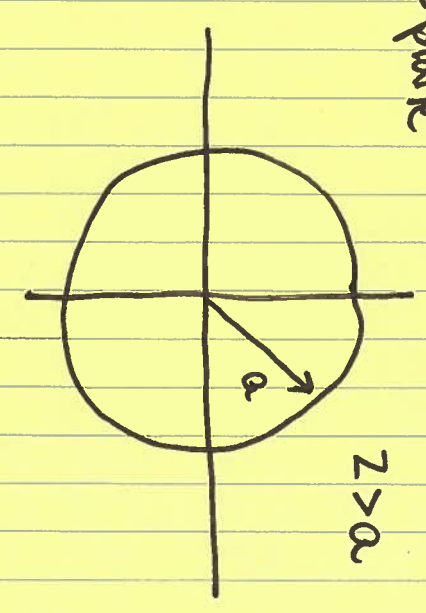
Use summation formula:

$$1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

$$X(z) = \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1-az^{-1}}$$

when $az^{-1} < 1 \quad |z| < \frac{1}{a} \quad z > a$

z-plane



Ex:

$x(n) = -a^n u(-n-1)$
anti-causal version of $a^n u(n)$

$$X(z) = \frac{1}{1-az^{-1}}$$

But...



Stability

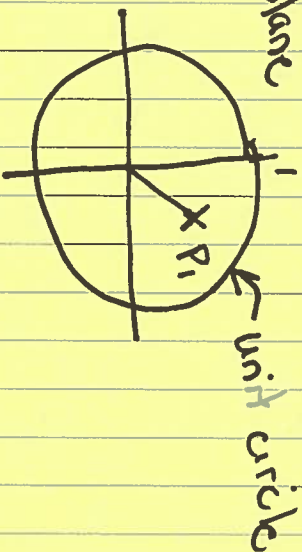
$$X(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots}$$

$$= \frac{A_1}{1 - p_1 z^{-1}} + \frac{A_2}{1 - p_2 z^{-1}} + \dots$$

$$X(z) = \frac{A_1}{1 - p_1 z^{-1}} \Rightarrow x(n) = [p_1^n u(n)] A_1$$

only exists if $|p_1| < 1$

z-plane

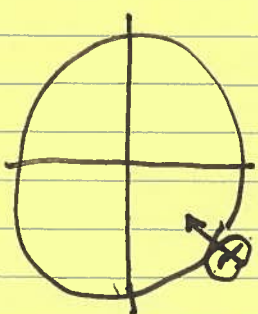


$|p_1| < 1 \Rightarrow$ stable

$$\sum_{n=0}^{\infty} |x(n)| < \infty$$

Note: the zeros of a transfer function
do not influence stability.

what do they influence?
phase.



$$X(n) = \delta(n) \quad Z[X(n)] = X(z) = 1$$

$$X(n) = a^n \quad X(z) = \frac{1}{1-z}$$

$$X(n) = \cos \omega n \quad X(z) = \frac{1}{1-z^2}$$

$$X(z) = \sum_{n=0}^{\infty} \cos \omega n \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2} [e^{j\omega n} + e^{-j\omega n}] z^{-n}$$

Note: $e^{j\omega n} = a^n$ where $a = e^{j\omega}$

