Lecture 14: Pole/Zero Designs

$$H(z) = \frac{G(1+bz^{-1})}{1-az^{-1}}$$ 1 zero, 1 pole

$$H(0) = \frac{G(1+b)}{1-a} \qquad H(\pi) = \frac{G(1-b)}{1+a}$$

$$\frac{H(\pi)}{H(0)} = \frac{(1-b)(1-a)}{(1+b)(1+a)}$$

two degrees of freedom (a, b)

two unknowns $\Rightarrow$ two equations

$\Rightarrow$ two constraints

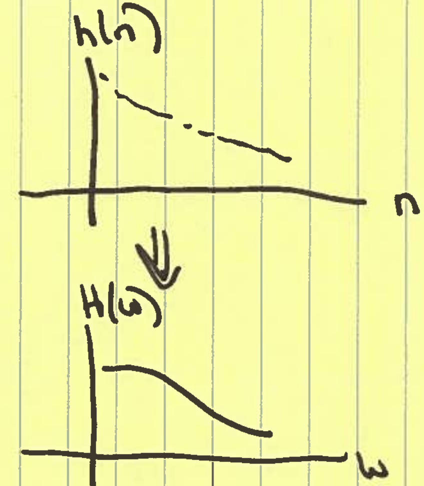
$\frac{H(\pi)}{H(0)}$ is one constraint

speed of response of the filter

$a = e^{1/n_{eff}}$ second constraint

controls the length of the impulse response

but, one pole filter have a constrained impulse response:



not much we can do with one pole!

Parametric Resonator

$$H(z) = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

a_1, a_2 are real numbers

(less multiply/adds)

poles must be complex conjugate pairs

$$H(z) = \frac{G}{(1 - z_1 z^{-1})(1 - z_1^* z^{-1})}$$

let $z_1 = R e^{j\omega_0}$ R is magnitude

$z_1^* = R e^{-j\omega_0}$ $e^{j\omega_0}$ is angle

$$H(z) = \frac{G}{(1 - R e^{j\omega_0} z^{-1})(1 - R e^{-j\omega_0} z^{-1})}$$

$$= \frac{G}{1 - (R e^{j\omega_0} + R e^{-j\omega_0}) z^{-1} + R^2 z^{-2}}$$

$$= \frac{G}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$a_1 = -2R\cos\omega_0 \quad a_2 = R^2$

$$R e^{j\omega_0} + R e^{-j\omega_0} = R(\cos\omega_0 + j\sin\omega_0) + R(\cos\omega_0 - j\sin\omega_0)$$

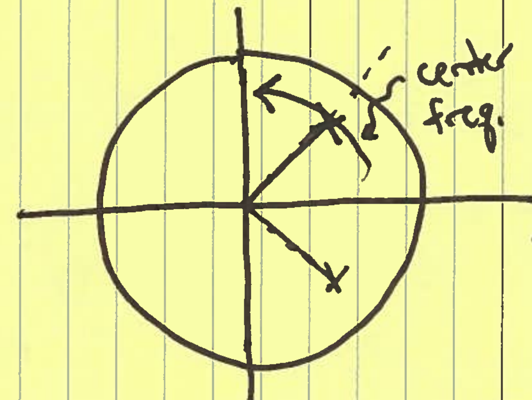
$$= 2R\cos\omega_0$$

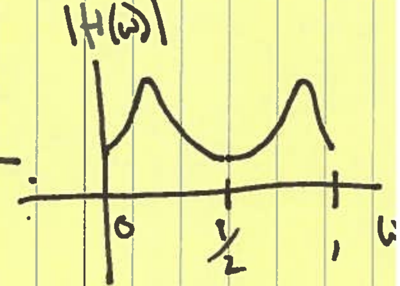
what about G?

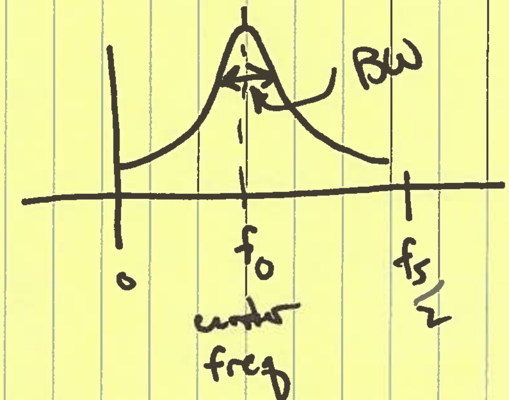
$$|H(\omega_0)| = 1$$

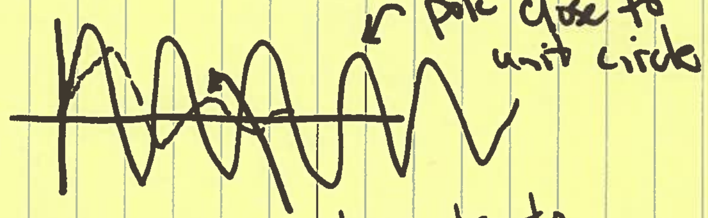
$$\left| \frac{G}{1 + a_1 e^{-j\omega_0} + a_2 e^{-j2\omega_0}} \right| = 1$$

$$G = (1 - R)\sqrt{1 - 2R\cos(2\omega_0) + R^2}$$

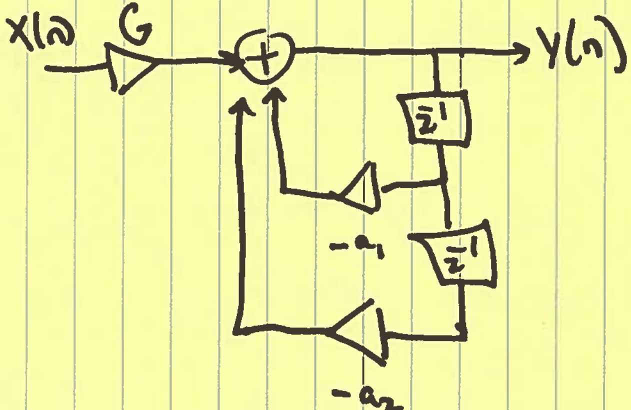








Implementation:



(Direct form)

Three design parameters:

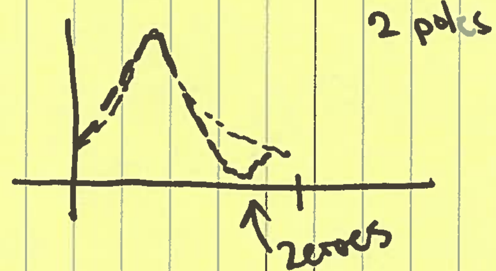
$G, f_0, BW \Rightarrow a_1, a_2, G$

more advanced:

$$H(z) = \frac{1 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$ (a, b are real)

poles are complex conjugates

zeroes are complex conjugates



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