

# Lecture 26

## FIR Filter Design

What is an FIR filter?

$$H(z) = 1 + b_1 z^{-1} + b_2 z^{-2} + \dots$$

$$\frac{Y(z)}{X(z)} = 1 + b_1 z^{-1} + \dots$$

$$Y(z) = X(z) + b_1 z^{-1} X(z) + \dots$$

$$y(n) = x(n) + b_1 x(n-1) + \dots$$

The filter ~~is~~ Design problem:

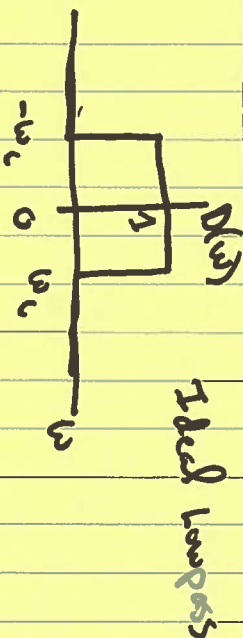
- (1) Specifications (e.g. cutoff freq.)
  - (2) Design a filter  $\Rightarrow$  produce a transfer function (e.g. FIR)
  - (3) Efficient implementation
  - (4) Fixed point implementation
- 1.2345  $\approx$  1.23

How do we design a filter?

Generate a transfer function.

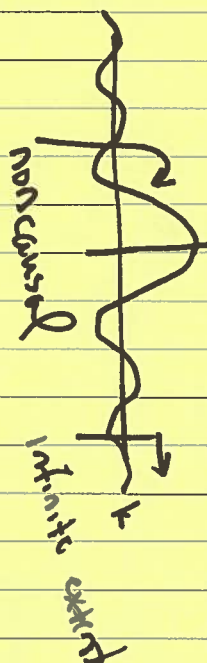
one approach: leverage analog filter design

Window method: Ideal Filters



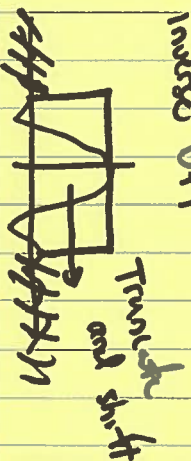
$\{1, 1, 1, 1\}$

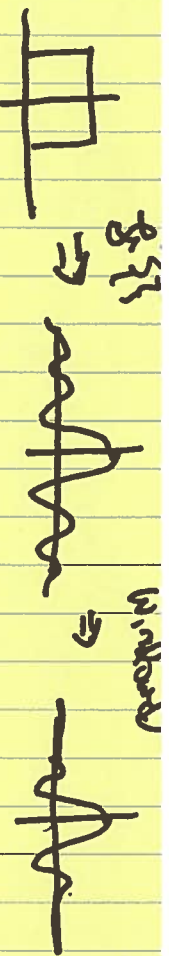
$$D(\omega) = \frac{\sin(\omega k)}{k} \quad -\infty < \omega < \infty$$



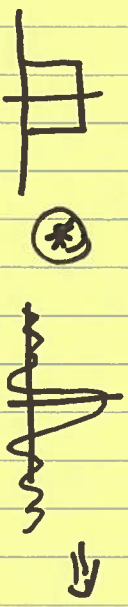
Simple strategy:

- (1) Take the inverse DFT
- (2) Window:





Gibbs Phenomenon:



Given we want an N-point FIR,  
 is this the best we can do?

Spec:

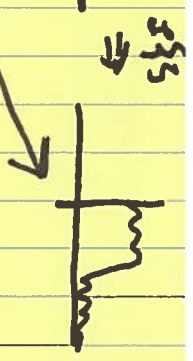


Constraint: N-pt. FIR filter  
 linear phase?

Is the rectangular window optimal?  
 could we use a Hamming window?

What is the best window function?

We want small ripple in the freq. domain  
 and narrow width in the time domain



Ripple!

Can a function be time-limited and  
 frequency limited? No!  
 Kaiser windows are generally regarded  
 as the closest approximation  
 to crossover filters

