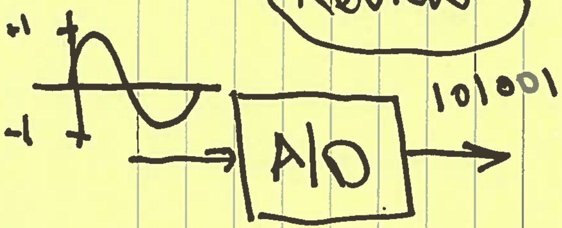
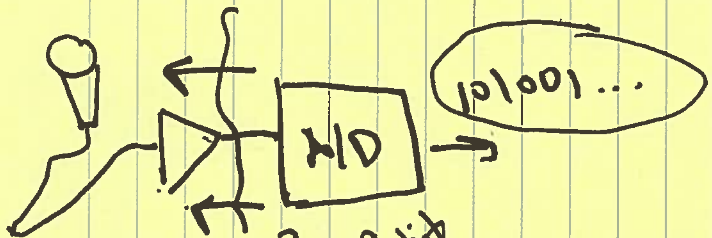
Review





\# of bits? 8 bit
14 bit
24 bit

linear converter 8 bit

1 V ⇒ 1111111 ⇒ 127

0 V ⇒ 0

-1 V ⇒ -128

$SNR \propto 6\,dB/bit \Rightarrow 48\,dB$

linear 16 bit converter ⇒ $SNR \propto 6 \cdot 16 \approx 96\,dB$

16 bits/sample

24 bit converter? $6 \cdot 24 \approx 144\,dB$

40 bits $\approx$ 240 dB

Discrete-time Systems

$x(nT)$: $[x[0], x[1], x[2]]$

$$x(n) \rightarrow \boxed{H} \rightarrow y(n)$$

$x(n)$: x_0, x_1, x_2

$x(0), x(1), x(2) \ldots$

Ex: $y(n) = 2x(n)$

$x(n) = [0, 1, 2, 3]$

$y(n) = [2\cdot 0, 2\cdot 1, 2\cdot 2, 2\cdot 3]$

$= [0, 2, 4, 6]$

Ex: $y(n) = x(n) + 0.5\, y(n-1)$

linear, constant coefficient difference equation

assume $y(n) = 0$, $n < 0$

Initial condition

$x(n) = [0, 1, 2, 3]$

$y(n) = [0, 1 + 0.5(0), \ldots]$

Ex: $y(n) = x^2(n)$ "nonlinear"

Linearity:

$$x(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$x(n) \longrightarrow \boxed{H} \longrightarrow y(n)$$

$$x_1(n) \Rightarrow y_1(n)$$

$$x_2(n) \Rightarrow y_2(n)$$

$\therefore$ if the system is linear and superposition holds:

$$y(n) = a_1 y_1(n) + a_2 y_2(n)$$

Ex. $y(n) = 2x(n)$

Yes!

$$y(n) = x^2(n)$$

$$= [a_1 x_1(n) + a_2 x_2(n)]^2$$

$$= a_1^2 x_1^2(n) + 2a_1 a_2 x_1(n) x_2(n) + a_2^2 x_2^2(n)$$

Note: $x(n) = a_1 x_1(n)$ $\quad$ $x(n) = a_2 x_2(n)$

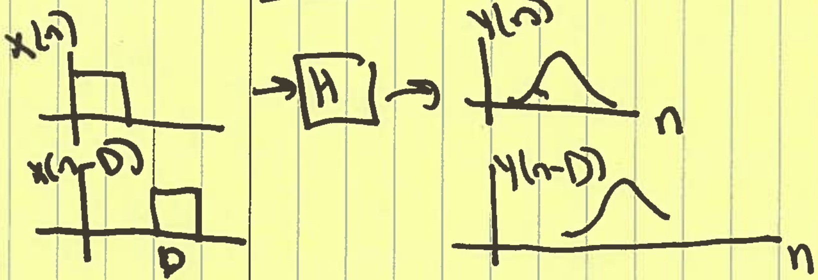
$y(n) = a_1^2 x_1^2(n)$ $\quad$ $y(n) = a_2^2 x_2^2(n)$

Real Amplifier: $y(n) = a_1 x(n) + a_2 x^2(n) + \ldots\ldots$

Time-Invariance:

$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n)$

$x(n-D) \rightarrow \boxed{H} \rightarrow y(n-D)$



Impulse Response:

$x(n) \rightarrow \boxed{H} \rightarrow y(n)$

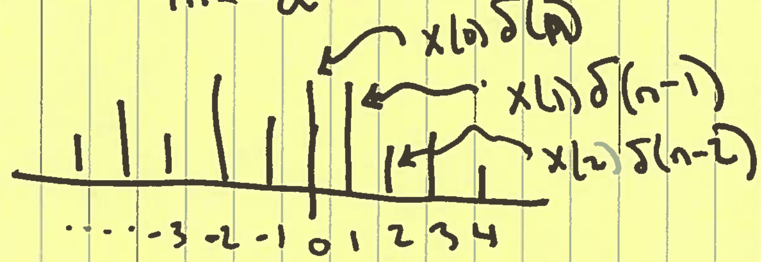
$$\delta(n) = \left\{ \begin{array}{ll} 1 & n=0 \\ 0 & \text{elsewhere} \end{array} \right\}$$

$\delta(n) \rightarrow \boxed{H} \rightarrow h(n)$

Linear time-invariant system:

$\delta(n-D) \rightarrow \boxed{H} \rightarrow h(n-D)$

$$x(n) = \sum_{m=-\infty}^{\infty} x(m)\,\delta(n-m)$$



$\delta(n) \rightarrow \boxed{H} \rightarrow h(n)$

$\delta(n-1) \rightarrow \boxed{H} \rightarrow h(n-1)$

$\vdots$

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\,h(n-m)$$

Convolution!

$$y(n) = \sum_{m=-N}^{N} x(m)\,h(n-m)$$

FIR filter

$$y(n) = \sum_{p=1}^{P} a_p\, x(n-p) + \sum_{q=1}^{Q} b_q\, y(n-q)$$

IIR filter