Name: _____

Problem	Points	Score
1(a)	15	
1(b)	15	
2(a)	15	
2(b)	15	
2(c)	5	
3(a)	15	
3(b)	10	
43c)	10	
Total	100	

Notes:

(1) The exam is closed books and notes except for four double-sided sheet of notes.

(2) Please indicate clearly your answer to the problem.

(3) If I can't read or follow your solution, it is wrong and no partial credit will be awarded.

Problem No. 1: Fast Fourier Transform

(15 pts) (1a) Direct multiplication of two complex numbers (a+jb)(c+jd) requires four real multiplications and two real additions. By properly arranging terms, show that it is possible to obtain the above multiplication using three real multiplications and five real additions.

(15 pts) (1b) Explain the key concepts behind the Fast Fourier Transform that allow a Discrete Fourier Transform to be computed faster and yet achieve the exact same result.

Problem No. 2: FIR Filter Design

The Hann window function can be written as: $w[n] = [0.5 - 0.5\cos(2\pi n / M)]w_R[n]$ where $w_R[n]$ is the rectangular window of length M+1.

(15 pts) (2a) Express the DTFT of w[n] in terms of the DTFT of $w_R[n]$.

(15 pts) (2b) Explain why the Hann window has the wider mainlobe but lower sidelobes than the rectangular window of the same length.

(5 pts) (2c) Explain why the width of the main lobe is important. Give an example.

Problem No. 3: Let x[n] be an input signal and h[n] denote the causal and stable IIR system. First, x[n] is filtered through h[n] to obtain output $y_1[n]$. Next, the flipped signal, x[-n], is filtered through h[n] to obtain $y_2[n]$. Finally, the output is computed by summing $y[n] = y_1[n]$ and $y_2[n]$.

(15 pts) (3a) Let $h_s[n]$ be the impulse response of the overall system (e.g., x[n] is the input, y[n] is the output). Derive an expression for $h_s[n]$ in terms of h[n].

(10 pts) (3b) Determine the frequency response of $h_s[n]$ and discuss any important properties you observe.

(10 pts) (3c) Will these properties hold if h[n] is an IIR filter? Explain the significance of your findings.