Basic Problems

20. (a) Solution:
Follow the signal flow, we have

$$y[n] = x[n] * h[n] + x[-n] * h[n] = x[n] * h_{\text{zp}}[n]$$

where

$$x[-n] * h[n] = \sum_{m=-\infty}^{\infty} x[-m]h[n-m] = \sum_{m=-\infty}^{\infty} x[m]h[n+m] = x[n] * h[-n]$$

Plug in the above result in, we have

$$y[n] = x[n] * h[n] + x[n] * h[-n] = x[n]\,(h[n] + h[-n])$$

thus,

$$h_{\text{zp}}[n] = h[n] + h[-n]$$

(b) Solution:
The frequency response is:

$$H_{\text{zp}}\big(e^{j\omega}\big) = H\big(e^{j\omega}\big) + H\left(e^{-j\omega}\right) = 2\text{Re}\big[H\big(e^{j\omega}\big)\big]$$

(c) tba.

21. (a) See plot below.

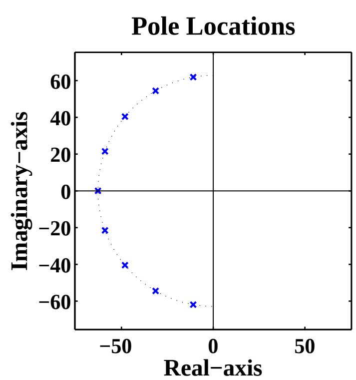

FIGURE 11.27: Pole locations of $H_c(s)$

(b) See plot below.

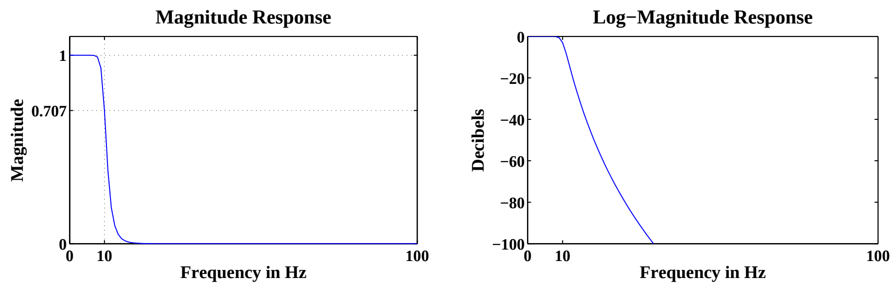

FIGURE 11.28: Magnitude and log-magnitude responses over $[0, 100]$ Hz range.

(c) Solution:
The frequencies are 14.68, 16.68, and 18.96 rad/s at which the attenuation is 30 dB, 40 dB, and 50 dB.

MATLAB script:

```
% P1121: Ninth order Butterworth low-pass filter
close all; clc
N = 9;
% Determine cutoff frequency
Hc = 10;
Omegac = Hc*2*pi;
% Determine zero locations
k = 1:N; thetak = pi/2+(2*k-1)*pi/2/N;
sigmak = Omegac*cos(thetak); Omegak = Omegac*sin(thetak);
sk = cplxpair(sigmak + 1j*Omegak);
% Compute system function
C = Omegac^N;
D = real(poly(sk));
Fmax = 100;
F = linspace(0,Fmax,101);
Om = F*2*pi;
H = freqs(C,D,F*2*pi);
Hmag = abs(H); Hpha = angle(H);
Hdb = 20*log10(Hmag);
Hgdl = -diff(unwrap(Hpha))./diff(Om); Hgdl = [Hgdl,Hgdl(end)];
% Part c:
A = [3 30 40 50];
delta = (10.^(-A/20)).^2;
disp('Frequencies where attenuation is 3db, 30db, 40db, 50db')
```

```
F3 = (1./delta-1).^(1/2/N)*Hc,
%% Plot:
hfa = figconfg('P1121a','small');
plot(sigmak,Omegak,'bx','linewidth',1.5); hold on;
ff = 1.2; pc = ff*Omegac;
plot([-pc,pc],[0,0],'k','linewidth',0.75);
plot([0,0],[-pc,pc],'k','linewidth',0.75);
plot(Omegac*cos(0.5*pi*[1:0.01:3]),Omegac*sin(0.5*pi*[1:0.01:3]),'k:');
axis([-pc,pc,-pc,pc]);
axis square;
xlabel('Real-axis','fontsize',LFS);
ylabel('Imaginary-axis','fontsize',LFS);
title('Pole Locations','fontsize',TFS);

hfb = figconfg('P1121b','small');
plot(F,Hmag,'b','linewidth',1);
axis([0,Fmax,0,1.1]);
xlabel('Frequency in Hz','fontsize',LFS);
ylabel('Magnitude','fontsize',LFS);
title('Magnitude Response','fontsize',TFS);
set(gca,'xtick',[0,Hc,Fmax]);
set(gca,'ytick',[0,0.707,1]); grid;

hfc = figconfg('P1121c','small');
plot(F,Hdb,'b','linewidth',1);
axis([0,Fmax,-100,0]);
xlabel('Frequency in Hz','fontsize',LFS);
ylabel('Decibels','fontsize',LFS);
title('Log-Magnitude Response','fontsize',TFS);
set(gca,'xtick',[0,Hc,Fmax]);
```

22. See plot below.

    MATLAB script:

```
% P1122: Analog Butterworth lowpass filter design
clc; close all;
% Given Design Parameters
Omegap = 10; Omegas = 15; Ap = 0.5; As = 40;
% Analog Design Parameters (Eq. 10.9)
epsilon = sqrt(10^(0.1*Ap)-1);
```

```
xlabel('\omega/\pi','fontsize',LFS);
ylabel('Decibels','fontsize',LFS);
title('Log-Magnitude Response','fontsize',TFS);
set(gca,'xtick',[0,omegap,omegas,pi]/pi);
set(gca,'ytick',[-80,-As,-Ap]); grid; box off;

hfb = figconfg('P1130b','small'); % Phase Plot
plot(om/pi,Hpha,'b','linewidth',1);
xlabel('\omega/\pi','fontsize',LFS);
ylabel('rad','fontsize',LFS);
title('Phase Response','fontsize',TFS);
set(gca,'xtick',[0,omegap,omegas,pi]/pi);

hfc = figconfg('P1130c','small'); % Group-Delay Plot in Samples
plot(om/pi,Hgdl,'b','linewidth',1);
xlabel('\omega/\pi','fontsize',LFS);
ylabel('Samples','fontsize',LFS);
title('Group Delay Response','fontsize',TFS);
set(gca,'xtick',[0,omegap,omegas,pi]/pi);

hfd = figconfg('P1130d','small'); % Impulse Response Plots
stem(n*Td,h/Td,'filled','markersize',3,'color','r'); hold on;
plot(t,hc,'b','linewidth',1);
xlabel('Time in seconds','fontsize',LFS);
ylabel('Amplitude','fontsize',LFS);
title('Impulse Responses','fontsize',TFS);
```

31. See plot below.

    MATLAB script:

```
% P1131: Lowpass filter design by Chebyshev I and tranformation
%        using bilinear method
clc; close all;
% Given Design Parameters
omegap = 0.4*pi; omegas = 0.55*pi; Ap = 0.5; As = 50;
% Analog Design Parameters (Eq. 10.9)
epsilon = sqrt(10^(0.1*Ap)-1); A = 10^(0.05*As);
Rp = 1/sqrt(1+epsilon^2);
%% Step by Step Impulse Invariance Design
% Step-1: Choose Td
```

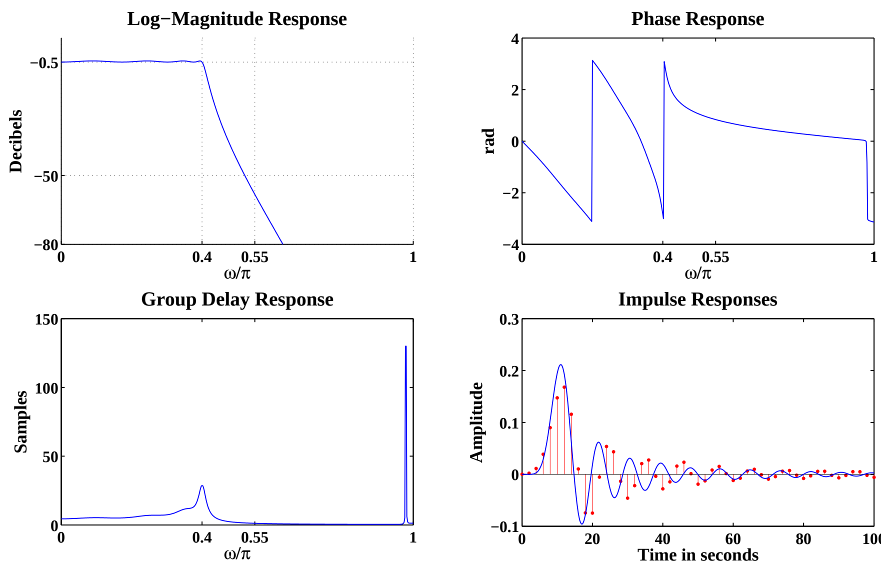

FIGURE 11.42: Plots of log-magnitude, phase, group, and impulse responses.

```
Td = 2;
% Step-2: Compute Analog Edge Frequencies
Omegap = 2/Td*tan(omegap/2); Omegas = 2/Td*tan(omegas/2);
% Step-3: Design Analog Chebyshev I Approximation
[N,Omegac] = cheb1ord(Omegap,Omegas,Ap,As,'s');
[C,D] = cheby1(N,Ap,Omegac,'s');
% Step-4: Obtain Digital Chebyshev I Filter
[B,A] = bilinear(C,D,1/Td);
% Plotting Parameters and Filter Responses
om = linspace(0,1,501)*pi; H = freqz(B,A,om);
Hmag = abs(H); Hpha = angle(H); Hdb = 20*log10(Hmag);
Hgdl = -diff(unwrap(Hpha))./diff(om); Hgdl = [Hgdl,Hgdl(end)];
Hgdl = medfilt1(Hgdl,3);
NN = 50; n = 0:NN; x = (n==0); h = filter(B,A,x);
t = linspace(0,NN*Td,501); hc = impulse(C,D,t);
% Exact Frequency:
ind = find(Hdb < -Ap,1,'first'); om1 = om(ind)/pi; % Exact Passband Edge
ind = find(Hdb < -As,1,'first'); om2 = om(ind)/pi; % Exact Stopband Edge
%% Design Plots
hfa = figconfg('P1131a','small'); % Log-Magnitude Response in dB
plot(om/pi,Hdb,'b','linewidth',1); axis([0,1,-80,10]);
```

```
xlabel('\omega/\pi','fontsize',LFS);
ylabel('Decibels','fontsize',LFS);
title('Log-Magnitude Response','fontsize',TFS);
set(gca,'xtick',[0,omegap,omegas,pi]/pi);
set(gca,'ytick',[-80,-As,-Ap]); grid; box off;

hfb = figconfg('P1131b','small'); % Phase Plot
plot(om/pi,Hpha,'b','linewidth',1);
xlabel('\omega/\pi','fontsize',LFS);
ylabel('rad','fontsize',LFS);
title('Phase Response','fontsize',TFS);
set(gca,'xtick',[0,omegap,omegas,pi]/pi);

hfc = figconfg('P1131c','small'); % Group-Delay Plot in Samples
plot(om/pi,Hgdl,'b','linewidth',1);
xlabel('\omega/\pi','fontsize',LFS);
ylabel('Samples','fontsize',LFS);
title('Group Delay Response','fontsize',TFS);
set(gca,'xtick',[0,omegap,omegas,pi]/pi);

hfd = figconfg('P1131d','small'); % Impulse Response Plots
stem(n*Td,h/Td,'filled','markersize',3,'color','r'); hold on;
plot(t,hc,'b','linewidth',1);
xlabel('Time in seconds','fontsize',LFS);
ylabel('Amplitude','fontsize',LFS);
title('Impulse Responses','fontsize',TFS);
```

32. See plot below.

    MATLAB script:

```
% P1132: Lowpass filter design by Chebyshev II and tranformation
%        using bilinear method
clc; close all;
% Given Design Parameters
omegap = 0.2*pi; omegas = 0.4*pi; Ap = 1; As = 50;
% Analog Design Parameters (Eq. 10.9)
epsilon = sqrt(10^(0.1*Ap)-1); A = 10^(0.05*As);
Rp = 1/sqrt(1+epsilon^2);
%% Step by Step Impulse Invariance Design
% Step-1: Choose Td
```