

2. (a) Solution:

$$H(e^{j\omega}) = \frac{b}{1 + 0.81e^{-2j\omega}}$$

(b) Solution:

$$H(e^{j\omega}) = \frac{b}{1 + 0.81 \cos 2\omega - j0.81 \sin 2\omega}$$

$$\begin{aligned} |H(e^{j\omega})| &= \frac{|b|}{\sqrt{(1 + 0.81 \cos 2\omega)^2 + (0.81 \sin 2\omega)^2}} \\ &= \frac{|b|}{\sqrt{1 + 0.81^2 + 2 * 0.81 \cos 2\omega}} \end{aligned}$$

$$\begin{aligned} \max |H(e^{j\omega})| &= \frac{|b|}{\sqrt{1 + 0.81^2 - 2 * 0.81}} = 1 \\ |b| &= 0.19 \end{aligned}$$

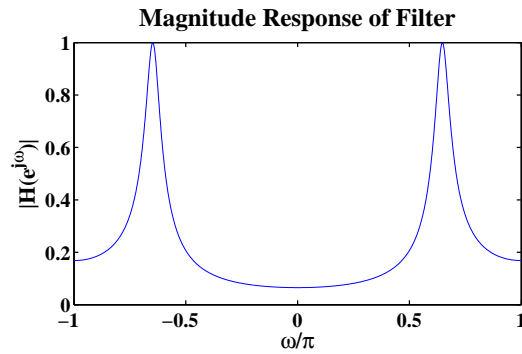


FIGURE 5.7: Magnitude response of the system.

(c) See plot below.

(d) Solution:

$$x[n] = 2 \cos(0.5\pi n + 60^\circ) = e^{j\frac{\pi}{3}} e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{3}} e^{-j\frac{\pi}{2}n}$$

$$\begin{aligned} y[n] &= e^{j\frac{\pi}{3}} e^{j\frac{\pi}{2}n} H(e^{j\frac{\pi}{2}}) + e^{-j\frac{\pi}{3}} e^{-j\frac{\pi}{2}n} H(e^{j-\frac{\pi}{2}}) \\ &= 2 \cos\left(\frac{\pi}{2}n + \frac{\pi}{3}\right) \end{aligned}$$

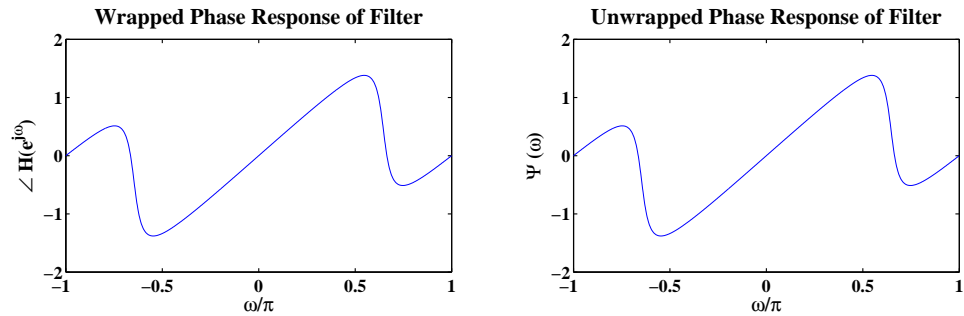
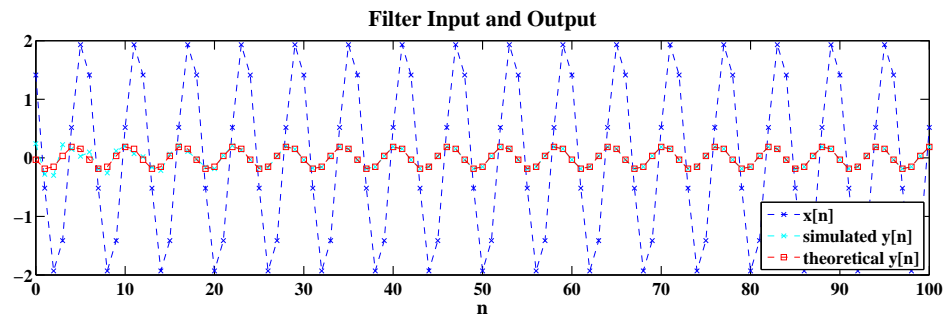


FIGURE 5.8: Wrapped and the unwrapped phase responses of the system.

FIGURE 5.9: MATLAB verification of the steady-state response to $x[n]$.

(e) See plot below.

MATLAB script:

```
% P0502: Plot magnitude and phase response
close all;clc
b = sqrt(1+0.8^2+0.81^2-2*0.81-0.8^2*1.81^2/2/2/0.81);
a = [1 0.8 0.81];
w = linspace(-pi,pi,1000);
H = freqz(b,a,w);
H_mag = abs(H);
H_phase = angle(H);
H_phase_unwrap = unwrap(H_phase);
n = 0:100;
xn = 2*cos(pi*n/3+pi/4);
yn = filter(b,a,xn);
yn_ref = 2*0.0577*cos(pi*n/3+pi/4)-2*0.0809*sin(pi*n/3+pi/4);
```

```

%% Plot:
hfa = figconfg('P0502a','small');
plot(w/pi,H_mag)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('|H(e^{j\omega})|','fontsize',LFS)
title('Magnitude Response of Filter','fontsize',TFS)
hfb = figconfg('P0502b','long');
subplot(121)
plot(w/pi,H_phase)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\angle H(e^{j\omega})','fontsize',LFS)
title('Wrapped Phase Response of Filter','fontsize',TFS)
subplot(122)
plot(w/pi,H_phase_unwrap)
xlabel('\omega/\pi','fontsize',LFS)
ylabel('\Psi (\omega)','fontsize',LFS)
title('Unwrapped Phase Response of Filter','fontsize',TFS)
hfc = figconfg('P0502c','long');
plot(n,xn,'--xb',n,yn,'--xc',n,yn_ref,'--sr')
legend('x[n]','simulated y[n]','theoretical y[n]','Location','Southeast')
xlabel('n','fontsize',LFS)
title('Filter Input and Output','fontsize',TFS)

```

3. (a) Solution:

$$\begin{aligned}
 |H(e^{j2\pi F})| &= \frac{0.2}{\sqrt{(1 - 0.8 \cos \omega)^2 + (0.8 \sin \omega)^2}} \\
 &= \frac{0.2}{\sqrt{1.64 - 1.6 \cos \omega}}, \quad \omega = 2\pi F/F_s
 \end{aligned}$$

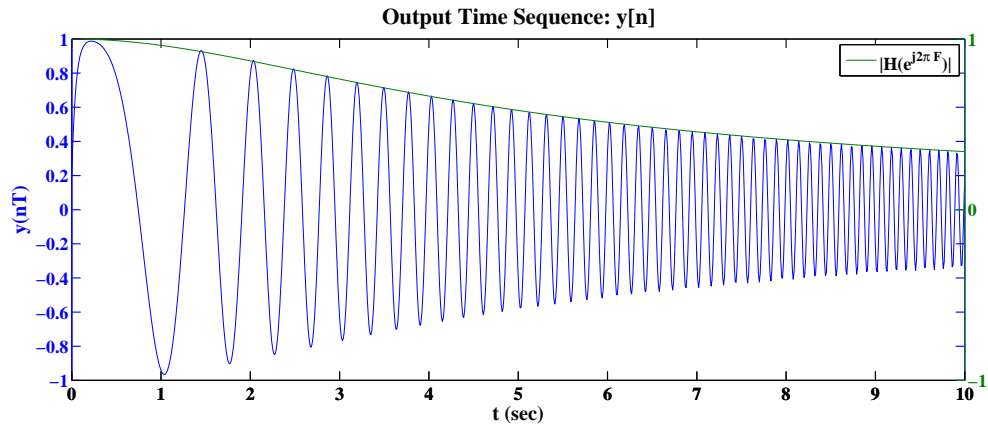
(c) See plot below.

MATLAB script:

```

% P0503: Linear FM signal
close all; clc
%% Specification:
B = 10;
Fs = 100;
tau = 10;
N = tau*Fs;

```

FIGURE 5.12: Plot of $y[n] = y(nT)$ over $0 \leq t \leq \tau$ sec.

```

xlabel('F (Hz)', 'fontsize', LFS)
ylabel('|H(e^{j2\pi F})|', 'fontsize', LFS)
title('Magnitude Response of Filter', 'fontsize', TFS)
hfb = figconfg('P0503b', 'long');
plot(n/Fs, xn)
xlabel('t (sec)', 'fontsize', LFS)
ylabel('x(nT)', 'fontsize', LFS)
title('Input Time Sequence: x[n]', 'fontsize', TFS)
hfc = figconfg('P0503c', 'long');
[AX H1 H2] = plotyy(n/Fs, yn, n/Fs, H_mag);
set(AX(2), 'ylim', [-1 1], 'YTick', -1:1)
xlabel('t (sec)', 'fontsize', LFS)
ylabel('y(nT)', 'fontsize', LFS)
title('Output Time Sequence: y[n]', 'fontsize', TFS)
legend('|H(e^{j2\pi F})|')

```

4. (a) Solution:

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$$

$$Y(z) = \frac{1}{1 - \frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - \frac{1}{2}e^{-j\omega}}{1 - \frac{1}{3}e^{-j\omega}}$$

which is unique.

(b) Solution:

$$H(e^{j\frac{\pi}{3}}) = 2$$

Hence, the frequency response function exists but not unique.

(c) $x[n] = \frac{\sin \pi n/4}{\pi n} \xrightarrow{\mathcal{H}} y[n] = \frac{\sin \pi n/2}{\pi n}$

Solution:

$$y[n] = 2x[2n]$$

which is time-varying.

(d) Solution:

$$y[n] = x[n] - x[n - 1]$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = 1 - e^{-j\omega}$$

which is unique.

5. (a) See plot below.

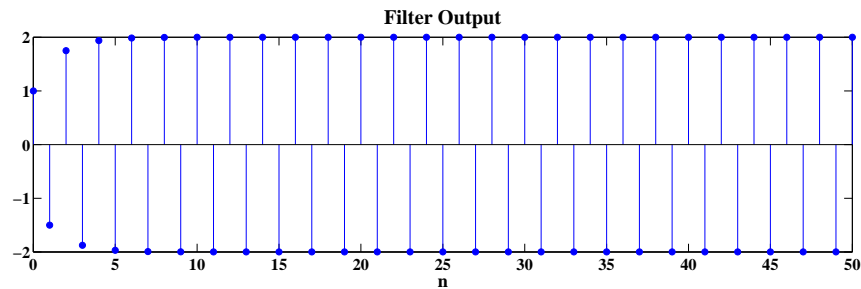


FIGURE 5.13: System response to the input $x[n] = (-1)^n u[n]$.

(b) See plot below.

(c) See plot below.

MATLAB script:

```
% P0505: Checking whether a system is LTI
%       Compute impulse response and frequency response
close all; clc
%% Part a:
n = 0:50;
N = length(n);
xn1 = (-1).^n;
```

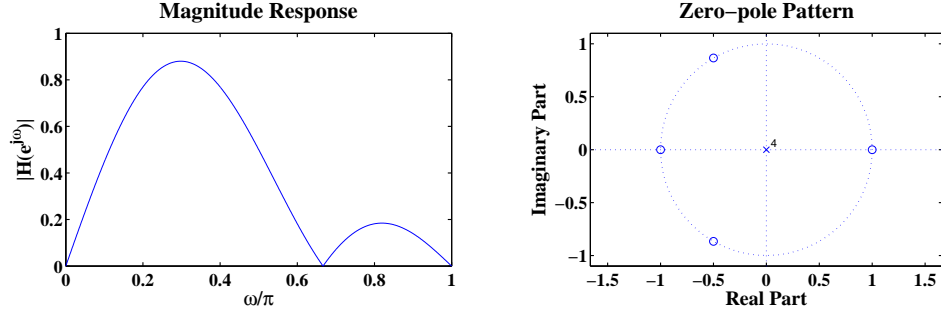


FIGURE 5.21: Magnitude response and pole-zero plot of $y[n] = \frac{1}{4}(x[n] + x[n-1]) - \frac{1}{4}(x[n-3] + x[n-4])$.

8. (a) Solution:

$$h_{bp}[n] = 2 \frac{\sin \omega_c (n - n_d)}{\pi (n - n_d)} \cos \omega_0 n \quad (5.72)$$

$$h_{lp}[n] = \frac{\sin \omega_c (n - n_d)}{\pi (n - n_d)} \quad (5.70)$$

Modulation Property:

$$x[n] \cos \omega_0 n = \frac{1}{2} X(e^{j(\omega + \omega_0)}) + \frac{1}{2} X(e^{j(\omega - \omega_0)})$$

$$H_{bp}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & \omega_\ell \leq |\omega| \leq \omega_h \\ 0, & \text{otherwise} \end{cases}$$

$$H_{lp}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & |\omega| \leq \omega_c \\ 0, & \omega_c < |\omega| \leq \pi \end{cases}$$

$$H_{bp}(e^{j\omega}) = H_{lp}(e^{j(\omega - \omega_0)}) + H_{lp}(e^{j(\omega + \omega_0)})$$

where $\omega_0 = \frac{\omega_\ell + \omega_h}{2}$ and $\omega_c = \frac{\omega_h - \omega_\ell}{2}$.

Hence,

$$h_{bp}[n] = 2h_{lp} \cos \omega_0 n = 2 \frac{\sin \omega_c (n - n_d)}{\pi (n - n_d)} \cos \omega_0 n$$

(b) Solution:

$$H_{bp}(e^{j\omega}) = H_{lp1}(e^{j\omega}) - H_{lp2}(e^{j\omega})$$

where

$$H_{lp1}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & |\omega| \leq \omega_h \\ 0, & \omega_h < |\omega| \leq \pi \end{cases}$$

$$H_{lp2}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & |\omega| \leq \omega_\ell \\ 0, & \omega_\ell < |\omega| \leq \pi \end{cases}$$

9. MATLAB function:

```
function [grp,omega] = mygrpdelay(b,a)
% Implement equation (5.89) to compute group delay
p = roots(a);
p = p(:);
N = size(p,1);
r = abs(p);
phi = angle(p);

z = roots(b);
z = z(:);
M = size(z,1);
q = abs(z);
theta = angle(z);

K = 1024;
omega = 2*pi*(0:K-1)/K;

r_epd = repmat(r,1,K);
phi_epd = repmat(phi,1,K);
q_epd = repmat(q,1,K);
theta_epd = repmat(theta,1,K);
temp1 = cos(repmat(omega,N,1)-phi_epd);
temp2 = cos(repmat(omega,M,1)-theta_epd);

grp = sum((r_epd.^2-r_epd.*temp1)./(1+r_epd.^2-2*r_epd.*temp1),1);
grp = -grp + sum((q_epd.^2-q_epd.*temp2)./(1+q_epd.^2-2*q_epd.*temp2),1);
```

Basic Problems

23. (a) Solution:
The frequency response is

$$H(e^{j\omega}) = \frac{b}{1 - 0.8e^{-j\omega} - 0.81e^{-2j\omega}}$$

- (b) Solution:
 $b = 0.1702$.

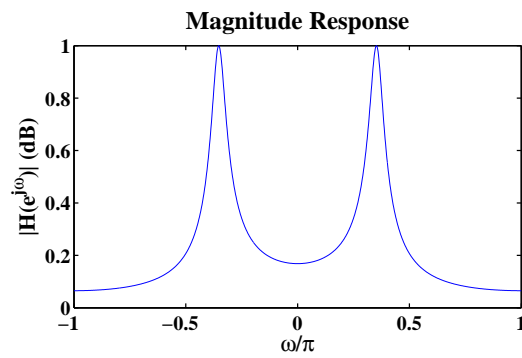


FIGURE 5.52: Magnitude response.

- (c) See plot below.

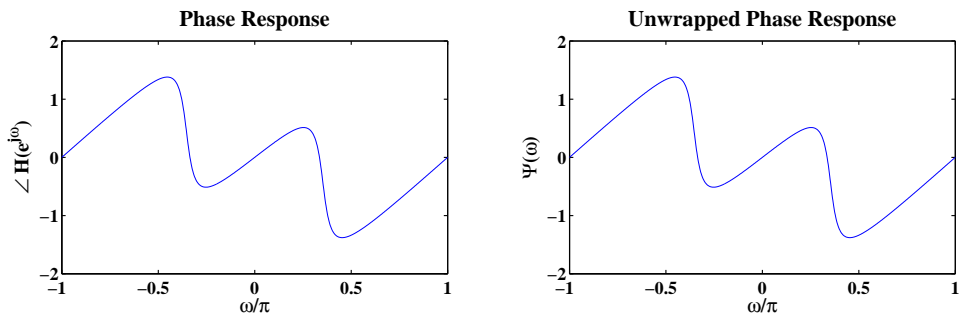


FIGURE 5.53: Wrapped and the unwrapped phase responses.

- (d) Solution:

$$x[n] = 2 \cos\left(\frac{\pi n}{3} + \frac{\pi}{4}\right) = e^{j\frac{\pi}{4}} e^{j\frac{\pi n}{3}} + e^{-j\frac{\pi}{4}} e^{-j\frac{\pi n}{3}}$$

$$\begin{aligned}
 y[n] &= e^{j\frac{\pi}{4}} e^{j\frac{\pi n}{3}} H(e^{j\frac{\pi}{3}}) + e^{-j\frac{\pi}{4}} e^{-j\frac{\pi n}{3}} H(e^{-j\frac{\pi}{3}}) \\
 &= 2 \times 0.0577 \cos\left(\frac{\pi n}{3} + \frac{\pi}{4}\right) - 2 \times 0.0809 \sin\left(\frac{\pi n}{3} + \frac{\pi}{4}\right)
 \end{aligned}$$

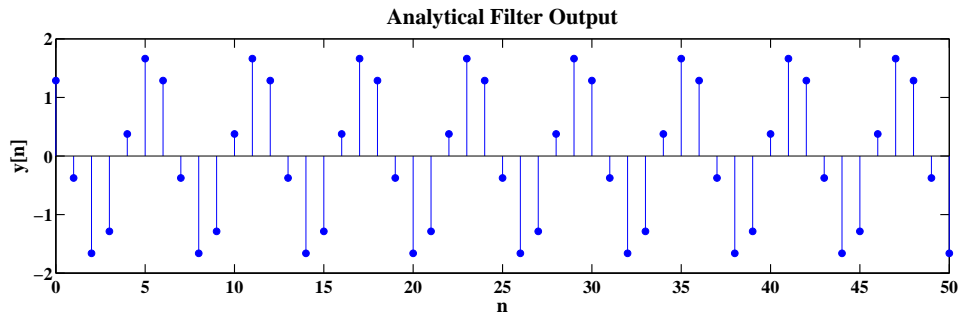


FIGURE 5.54: Analytical response $y[n]$ to the input $x[n] = 2 \cos(\pi n/3 + 45^\circ)$.

(e) See plot below.

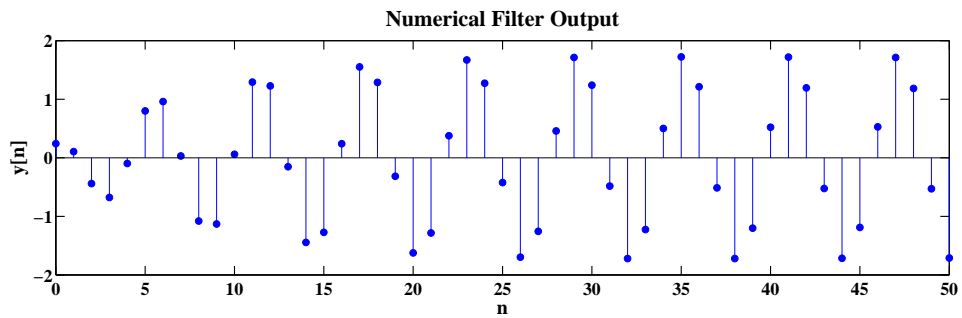


FIGURE 5.55: Numerical response $y[n]$ to the input $x[n] = 2 \cos(\pi n/3 + 45^\circ)$.

24. (a) Solution:

The frequency response is:

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{b}{1 + ae^{-2j\omega}} = \frac{b}{1 + a \cos 2\omega - aj \sin 2\omega}$$

The magnitude response is:

$$|H(e^{j\omega})| = \frac{|b|}{\sqrt{(1 + a \cos 2\omega)^2 + a^2 \sin^2 2\omega}} = \frac{|b|}{\sqrt{1 + a^2 + 2a \cos 2\omega}}$$

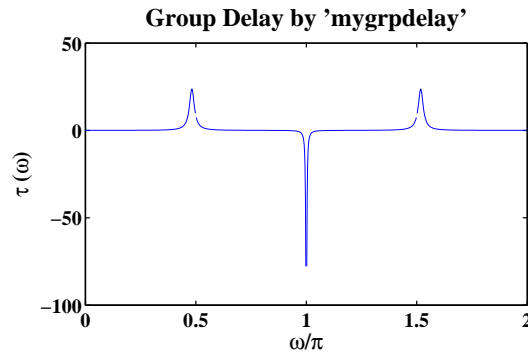


FIGURE 5.85: Group delay of the system using MATLAB function mygrpdelay.

```

q_epd = repmat(q,1,K);
theta_epd = repmat(theta,1,K);
temp1 = cos(repmat(omega,N,1)-phi_epd);
temp2 = cos(repmat(omega,M,1)-theta_epd);

grp = sum((r_epd.^2-r_epd.*temp1)./(1+r_epd.^2-2*r_epd.*temp1),1);
grp = -grp + sum((q_epd.^2-q_epd.*temp2)./(1+q_epd.^2-2*q_epd.*temp2),1);

```

35. (a) Solution:

$$H(z) = \frac{Y(z)}{X(z)} = 1 - \alpha \cdot z^{-1}$$

$$H(e^{j\omega}) = 1 - \alpha \cdot e^{-j\omega} = 1 - \alpha \cos \omega + j\alpha \sin \omega$$

$$\angle H(e^{j\omega}) = \tan^{-1} \frac{\alpha \sin \omega}{1 - \alpha \cos \omega}$$

$$\begin{aligned} \tau_{\text{gd}}(\omega) &= \frac{-d\angle H(e^{j\omega}) + 2k\pi}{d\omega} \\ &= -\frac{1}{1 + \left(\frac{\alpha \sin \omega}{1 - \alpha \cos \omega}\right)^2} \cdot \frac{\alpha \cos \omega (1 - \alpha \cos \omega) - \alpha \sin \omega \cdot \alpha \sin \omega}{(1 - \alpha \cos \omega)^2} \\ &= \frac{\alpha^2 - \alpha \cos \omega}{1 + \alpha^2 - 2\alpha \cos \omega} \end{aligned}$$

(b) Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \alpha \cdot z^{-1}}$$

$$H(e^{j\omega}) = \frac{1}{1 - \alpha \cdot e^{-j\omega}} = \frac{1}{1 - \alpha \cos \omega + j\alpha \sin \omega}$$

$$\angle H(e^{j\omega}) = -\tan^{-1} \frac{\alpha \sin \omega}{1 - \alpha \cos \omega}$$

$$\begin{aligned} \tau_{\text{gd}}(\omega) &= \frac{-d\angle H(e^{j\omega}) + 2k\pi}{d\omega} \\ &= \frac{1}{1 + \left(\frac{\alpha \sin \omega}{1 - \alpha \cos \omega}\right)^2} \cdot \frac{\alpha \cos \omega(1 - \alpha \cos \omega) - \alpha \sin \omega \cdot \alpha \sin \omega}{(1 - \alpha \cos \omega)^2} \\ &= \frac{-\alpha^2 + \alpha \cos \omega}{1 + \alpha^2 - 2\alpha \cos \omega} \end{aligned}$$

(c) Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 2\alpha \cos \phi z^{-1} + \alpha^2 z^{-2}}$$

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{1 - 2\alpha \cos \phi e^{-j\omega} + \alpha^2 e^{-2j\omega}} \\ &= \frac{1}{(1 - 2\alpha \cos \phi \cos \omega + \alpha^2 \cos 2\omega) + j(2\alpha \cos \phi \sin \omega - \alpha^2 \sin 2\omega)} \end{aligned}$$

$$\angle H(e^{j\omega}) = -\tan^{-1} \frac{2\alpha \cos \phi \sin \omega - \alpha^2 \sin 2\omega}{1 - 2\alpha \cos \phi \cos \omega + \alpha^2 \cos 2\omega}$$

$$\begin{aligned} \tau_{\text{gd}}(\omega) &= \frac{-d\angle H(e^{j\omega}) + 2k\pi}{d\omega} \\ &= \frac{(2\alpha \cos \phi \cos \omega - 2\alpha^2 \cos 2\omega)(1 - 2\alpha \cos \phi \cos \omega + \alpha^2 \cos 2\omega) - (2\alpha \cos \phi \sin \omega - \alpha^2 \sin 2\omega)(2\alpha \cos \phi \sin \omega - 2\alpha^2 \sin 2\omega)}{(2\alpha \cos \phi \sin \omega - \alpha^2 \sin 2\omega)^2 + (1 - 2\alpha \cos \phi \cos \omega + \alpha^2 \cos 2\omega)^2} \\ &= \frac{4\alpha^3 \cos \phi \cos \omega - 2\alpha^4 - 4\alpha^2 \cos^2 \phi + 2\alpha^3 \sin \omega \sin 2\omega + 2\alpha \cos \phi \cos \omega + 2\alpha^2 \cos \phi \cos \omega \cos 2\omega - 2\alpha^2 \cos 2\omega}{4\alpha^2 \cos^2 \phi + 1 + \alpha^4 - 4\alpha^3 \cos \phi \cos \omega - 4\alpha \cos \phi \cos \omega + 2\alpha^2 \cos 2\omega} \end{aligned}$$

46. (a) Solution:

The frequency response of the system is:

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-4j\omega}}{1 + 0.81e^{-2j\omega} + 0.6561e^{-4j\omega}}$$

(b) See plot below.

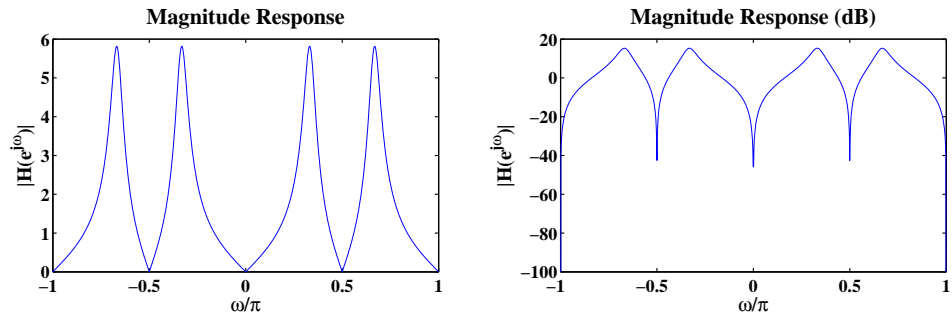


FIGURE 5.117: Magnitude and the dB-Gain responses of the system.

(c) See plot below.

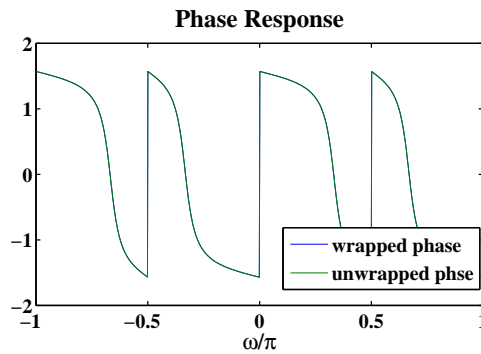


FIGURE 5.118: Wrapped and the unwrapped phase responses of the system.

(d) Solution:

$$x[n] = e^{j\frac{\pi}{3}} e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{3}} e^{-j\frac{\pi}{2}n} + \frac{1}{2j} e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{4}n} - \frac{1}{2j} e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{4}n}$$

$$\begin{aligned}
 y[n] &= e^{j\frac{\pi}{3}} e^{j\frac{\pi}{2}n} H(e^{j\frac{\pi}{2}}) + e^{-j\frac{\pi}{3}} e^{-j\frac{\pi}{2}n} H(e^{-j\frac{\pi}{2}}) + \frac{1}{2j} e^{-j\frac{\pi}{4}} e^{j\frac{\pi}{4}n} H(e^{j\frac{\pi}{4}}) - \frac{1}{2j} e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{4}n} H(e^{-j\frac{\pi}{4}}) \\
 &= \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j\right) \frac{1}{0.3439 - j0.81} e^{j\frac{\pi}{4}n} + \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j\right) \frac{1}{0.3439 + j0.81} e^{-j\frac{\pi}{4}n}
 \end{aligned}$$

(e) See plot below.

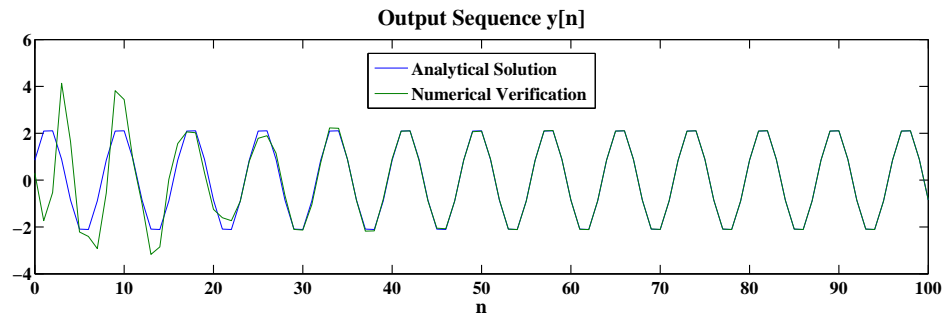


FIGURE 5.119: MATLAB verification of the steady-state response to $x[n]$.

47. (a) Solution:

$$\begin{aligned}
 c_k^{(x)} &= \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} = \frac{1}{5} \sum_{n=1}^4 (-0.5)^n e^{-j\frac{2\pi}{N}kn} \\
 &= \frac{1}{5} \frac{1 - (-1/2)^5}{1 + \frac{1}{2} e^{-j\frac{2\pi}{5}k}}, \quad 0 \leq k < 5
 \end{aligned}$$

(b) Solution:

$$c_k^{(y)} = c_k^{(x)} H(e^{j\frac{2\pi}{5}k})$$

(c) Solution:

$$y_{ss}[n] = \sum_{k=0}^4 c_k^{(x)} H(e^{j\frac{2\pi}{5}k}) e^{j\frac{2\pi}{5}kn}$$

(d) See plot below.

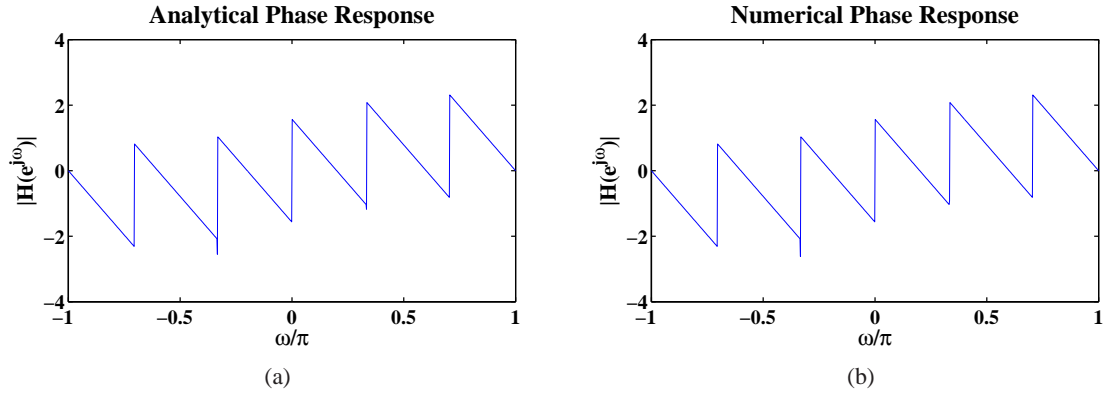


FIGURE 5.126: Phase responses plot (a) Analytical formula. (b) Numerical plot.

50. (a) See plot below.

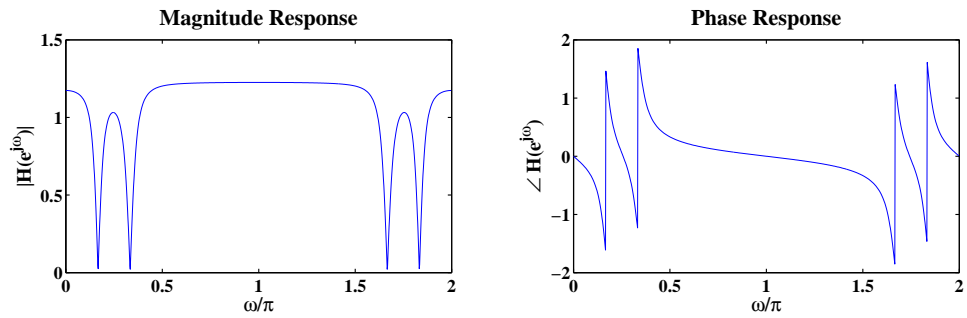


FIGURE 5.127: Magnitude and phase responses of the system using freqz.

(b) See plot below.

(c) See plot below.

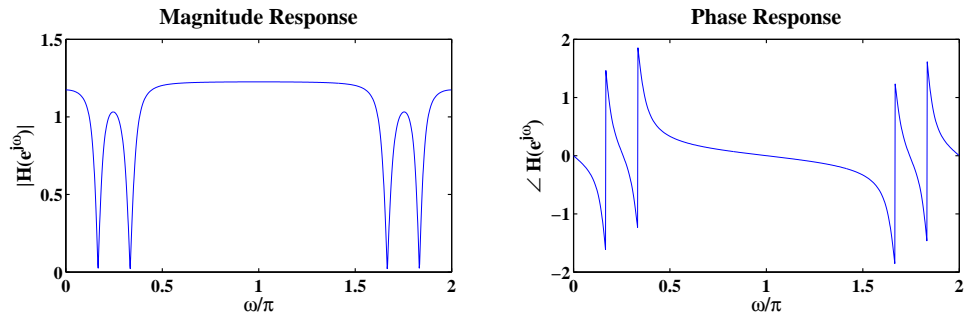


FIGURE 5.128: Magnitude and phase responses of the system using `freqz0`.

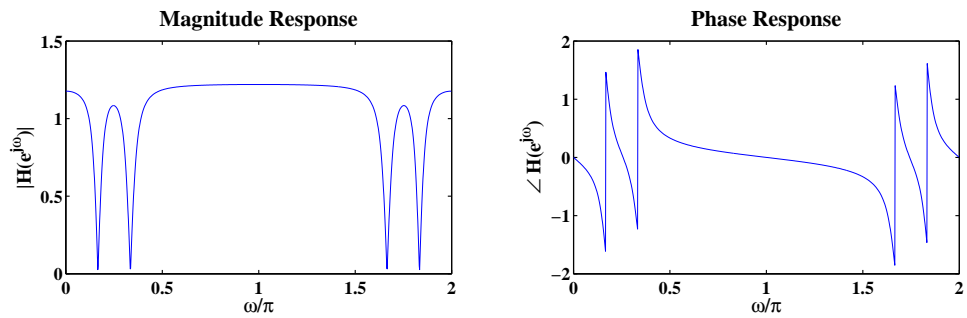


FIGURE 5.129: Magnitude and phase responses of the system using `[mag, pha, omega]=myfreqz(b, a)`.

51. (a) See plot below.
- (b) See plot below.
- (c) See plot below.

65. (a) Solution:

$$\text{zeros: } z_1 = e^{j\frac{\pi}{2}}, z_2 = e^{j\frac{3\pi}{2}}$$

$$\text{poles: } p_1 = re^{j\frac{2\pi}{3}}, p_2 = re^{-j\frac{2\pi}{3}}$$

The system function is:

$$H(z) = b_0 \frac{(1 - e^{j\frac{\pi}{2}} z^{-1})(1 - e^{j\frac{3\pi}{2}} z^{-1})}{(1 - re^{j\frac{2\pi}{3}} z^{-1})(1 - re^{-j\frac{2\pi}{3}} z^{-1})}$$

(b) See plot below.

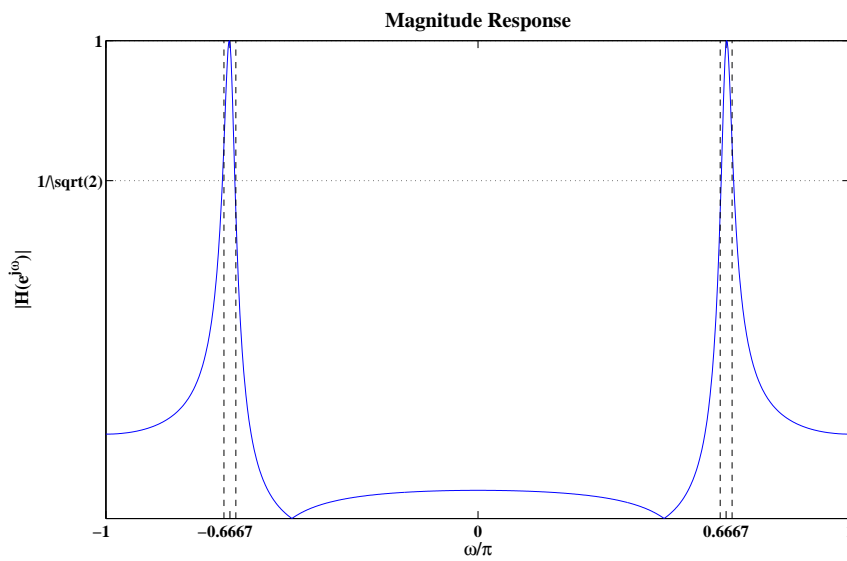


FIGURE 5.151: Magnitude response of the filter.

(c) See plot below.

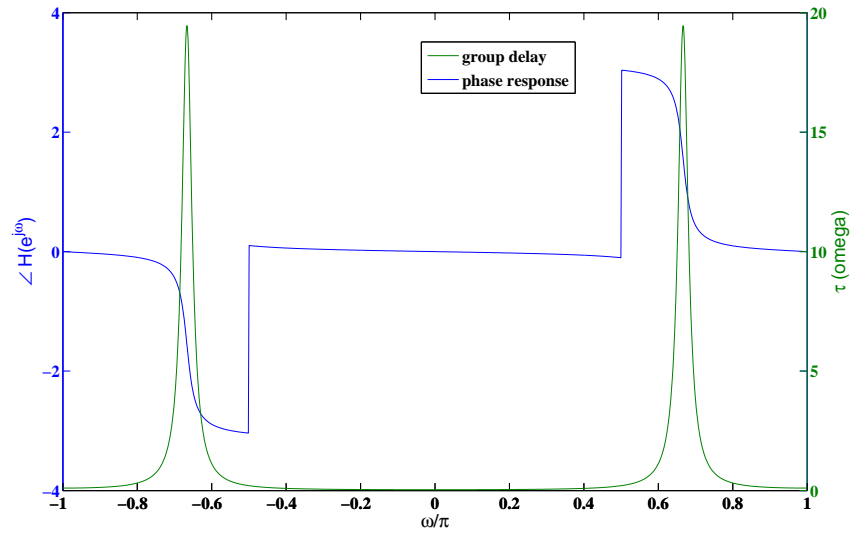


FIGURE 5.152: Phase and group-delay responses of the filter.

66. (a) Solution:

$$H(z) = H_{\min}(z) \cdot H_{\text{ap}}(z)$$

where

$$H_{\min}(z) = \frac{2.7 - 2.1z^{-1} + z^{-2}}{1 + 0.3126z^{-1} + 0.81z^{-2}}, \quad H_{\text{ap}}(z) = \frac{1 - 2.1z^{-1} + 2.7z^{-2}}{2.7 - 2.1z^{-1} + z^{-2}}$$

(b) See plot below.

(c) See plot below.

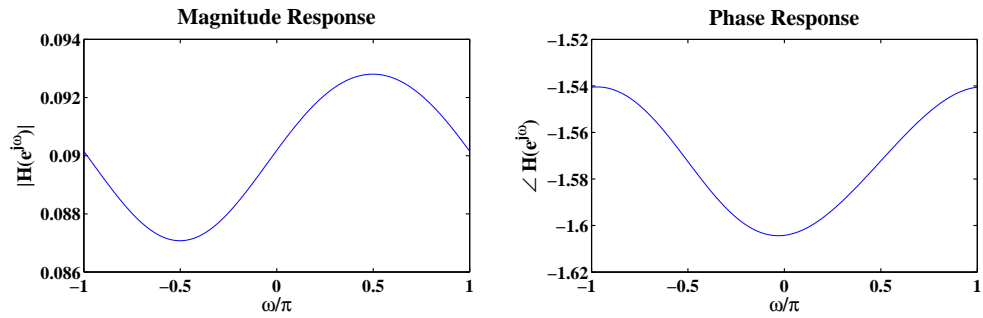


FIGURE 5.155: Magnitude and phase responses of the system.

$$y(t) = 4H(0) - \frac{3}{2}e^{j\frac{\pi}{3}}e^{j4\pi t}H(4\pi) - \frac{3}{2}e^{-j\frac{\pi}{3}}e^{-j4\pi t}H(-4\pi) + \frac{5}{2j}e^{j20\pi t}H(20\pi) - \frac{5}{2j}e^{-j20\pi t}H(-20\pi)$$

68. Solution:

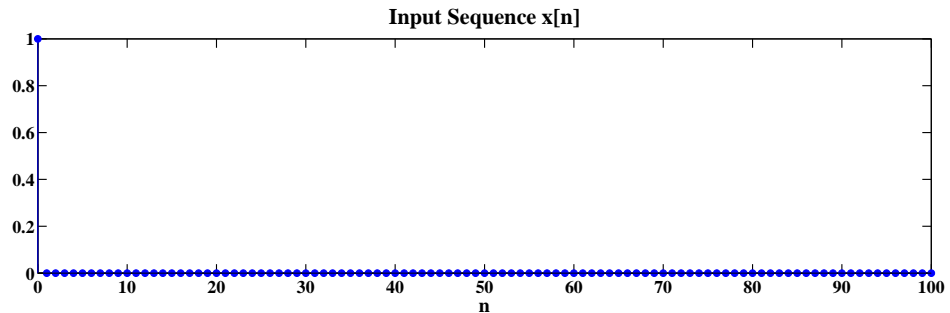
Choose $D = 5$.

FIGURE 5.156: Input sequence (i).

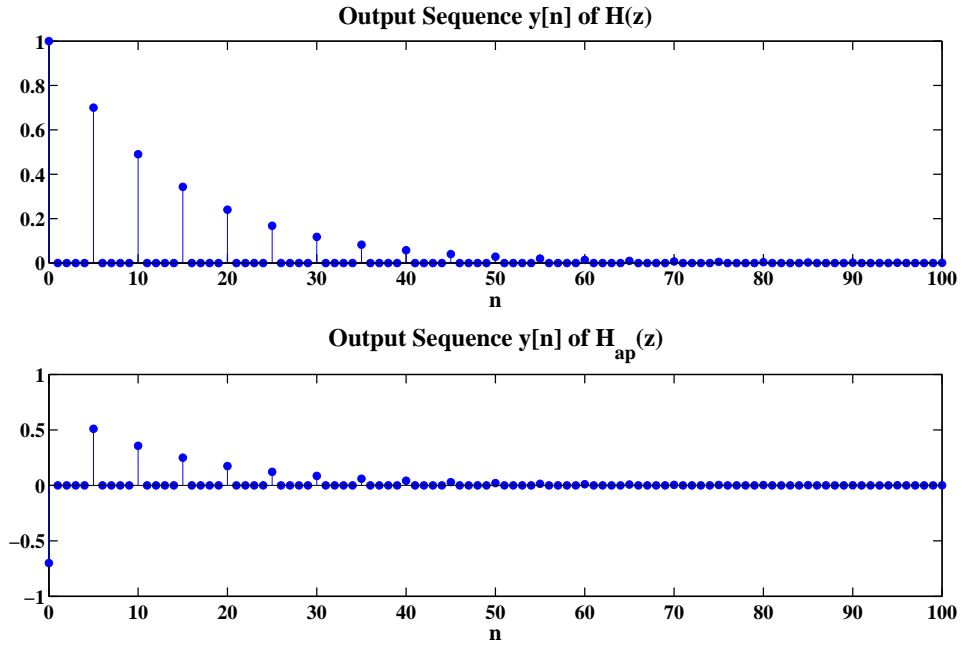


FIGURE 5.157: Output sequences to input (i).

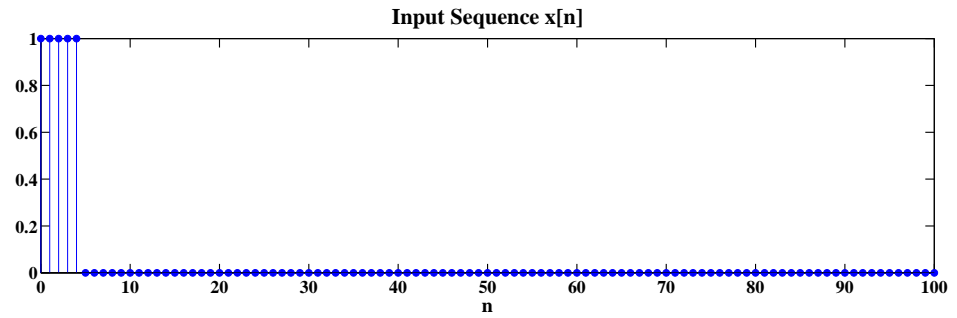


FIGURE 5.158: Input sequence (ii).

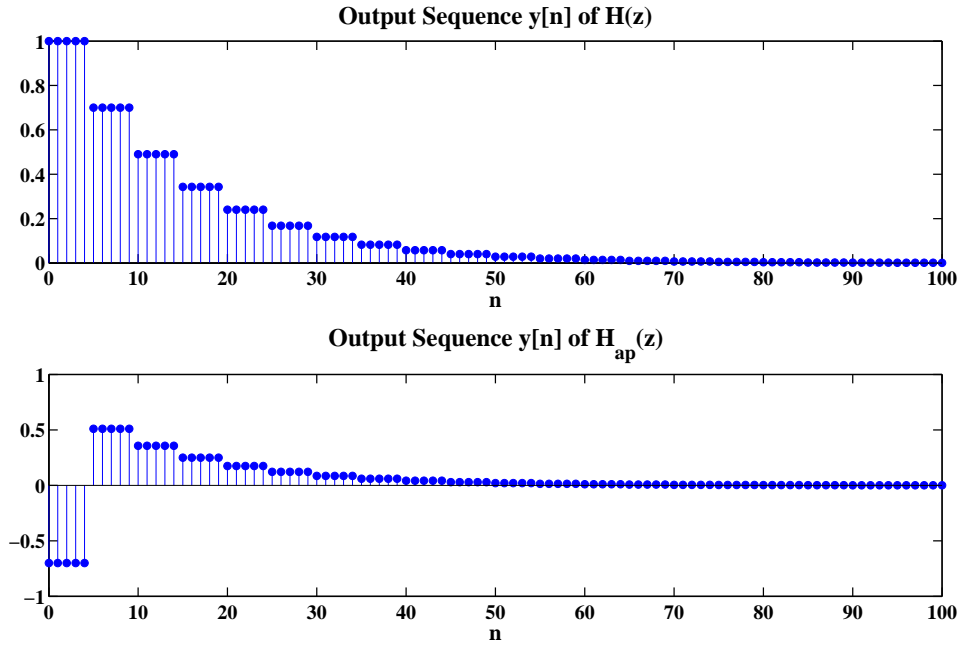


FIGURE 5.159: Output sequences to input (ii).

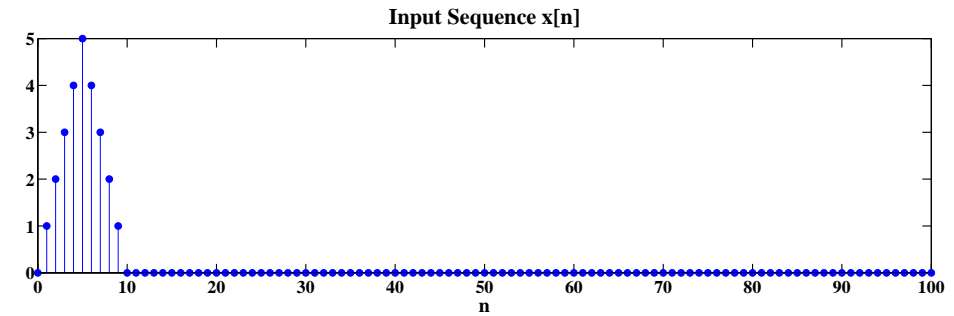


FIGURE 5.160: Input sequence (iii).

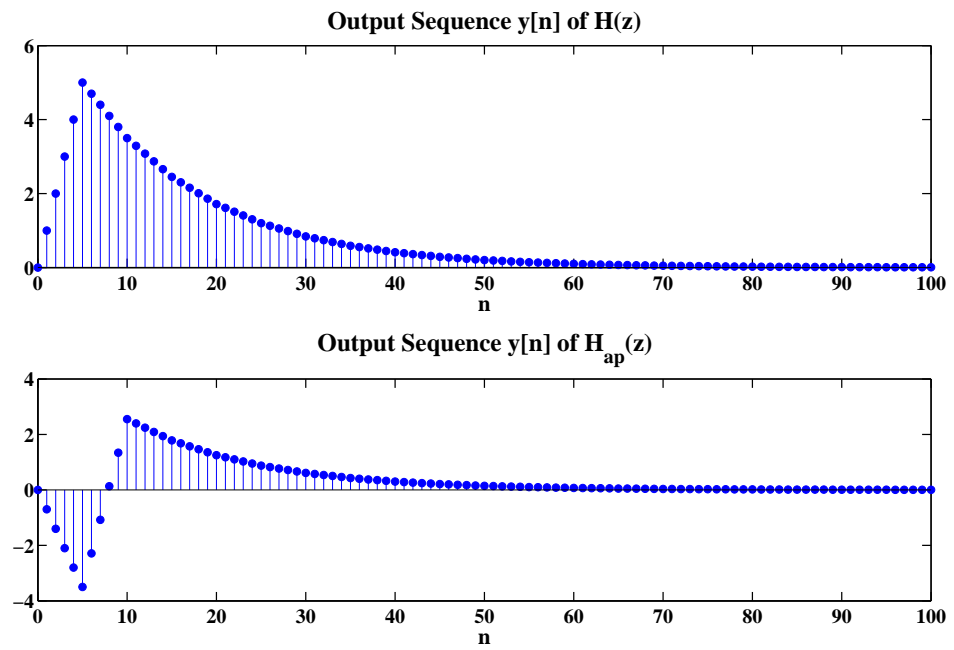


FIGURE 5.161: Output sequences to input (iii).

Review Problems

69. tba

70. (a) Solution:

$$G(z) = \frac{z^4 + 7z^3 + 20z^2 + 29z + 15}{z^5 + 4.1z^3 + 3.63z^2 + 2.015z + 0.63}$$

(b) tba.