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Exam 1 Redo
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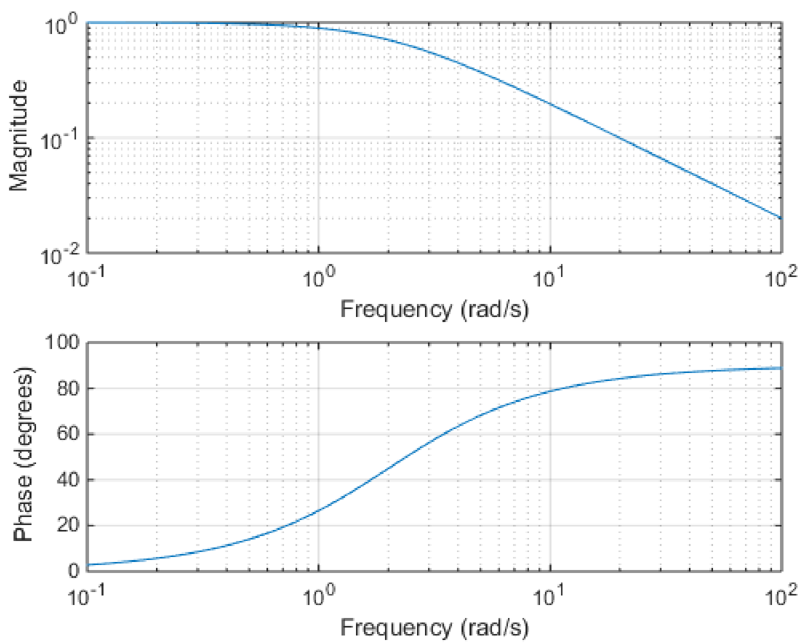
Problem 1

An LTI system is described by the difference equation:
$y[n] = 0.75y[n-1] - 0.125y[n-2] + x[n] - 0.25x[n-1]$
a) Derive an expression for the frequency response. Sketch your result - be specific as possible.

First we would take the z-transform of the difference equation given.

$$y[n] = 0.75y[n-1] - 0.125y[n-2] + x[n] - 0.25x[n-1]$$
$$y(z) = 0.75y(z)z^{-1} - 0.125y(z)z^{-2} + x(z) - 0.25x(z)z^{-1}$$
$$y(z) - 0.75y(z)z^{-1} + 0.125y(z)z^{-2} = x(z) - 0.25x(z)z^{-1}$$
$$y(z)(1 - 0.75z^{-1} + 0.125z^{-2}) = x(z)(1 - 0.25z^{-1})$$
$$\frac{y(z)}{x(z)} = \frac{1 - 0.25z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$
$$H(z) = \frac{1 - 0.25z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$
$$\boxed{H(e^{j\omega}) = \frac{1 - 0.25e^{-j\omega}}{1 - 0.75e^{-j\omega} + 0.125e^{-2j\omega}}}$$

To determine the response of the LTI system, the command $freqs$ can be used to plot the magnitude and phase of the transfer function. By plotting the magnitude response of the system, it is clear that the following transfer function produces the response of a low pass filter.



b) Derive an expression for the impulse response.

Starting with the Z-transform that was identified in the previous problem. The transform will be factored in such a way that the inverse transform will give the impulse response.

$$H(z) = \frac{1 - 0.25z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$H(z) = \frac{1 - 0.25z^{-1}}{(1 - 0.25z^{-1})(1 - 0.5z^{-1})}$$

From factoring the denominator, the numerator and a part of the denominator could then cancel out. The transform will then take the form of a common inverse Z-transform.

$$H(z) = \frac{1}{(1 - 0.5z^{-1})}$$

$$h[n] = \mathcal{L}_z^{-1}\left(\frac{1}{(1 - 0.5z^{-1})}\right)$$

$$\boxed{h[n] = (0.5)^n u[n]}$$

Problem 2

An 8-bit linear ADC has an input analog range of +/- 5V. The analog input signal is:

$x_c(t) = 2cos(200\pi t) + 3sin(500\pi t)$

The converter suplies data at a rate of 2048 bits/sec.

a) What is the quantizer step size?

The quantizer step size is found by the input analog range and the amount of bits that the ADC has at its disposal. Since the analog range is plus or minus 5 volts, this would give a value of 10 volts. Since the ADC has 8 bits this means that this is the value that 2 is raised to.

$$Q_{step} = \frac{D}{2^B}$$
$$where\ D = signal\ range\ and\ B = bits/sample$$
$$Q_{step} = \frac{10}{2^8}$$
$$\boxed{Q_{step} = 0.0390625}$$

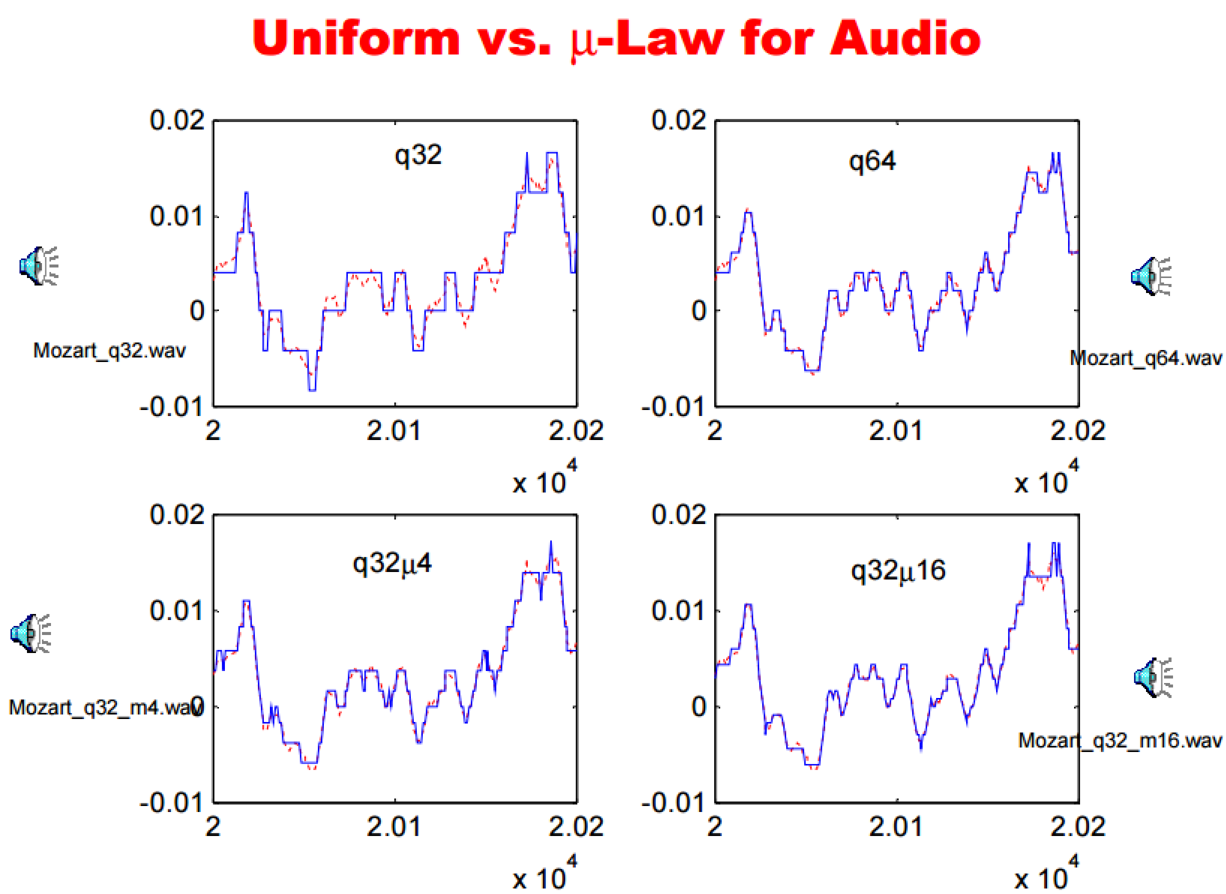
b) What is the SQNR in dβ?

In class we had derived the formula to solve for SQNR. The only value we would need to know to solve for the SQNR was the bits/sample term.

$$SQNR = 10log_{10}\left(\frac{\sigma_x^2}{\sigma_y^2}\right)$$
$$SQNR = 6.02B + 1.76$$
$$\boxed{SQNR = 49.92d\beta}$$

c) If the quantizer was an audio signal of the same range, how could you improve the performance of the ADC?

To improve the performance of the ADC using the same range for an audio signal, the quantization method should be altered. It currently was using the uniform quantization method however, to improve the performance switching to μ-law would have significant benefits. By using μ-law, the smaller parts of the audio signal would be accurately sampled. The higher parts of the audio signal would be misrepresented but its possible that much of these high parts would not be heard by the human ear anyway. During researching μ-law, I found a great comparison of using a uniform and μ-law quantizers. The μ-law can accurately replicate the signal with much less bits per sample, although near the higher parts it can get inaccurate. (Credits to Professor Yao Wang from Polytechnic University)



Problem 3

Compute and sketch the N-point DFT in the range of -(N-1)≤n≤(2N-1) for:

a) x[n]=δ[n],N=8

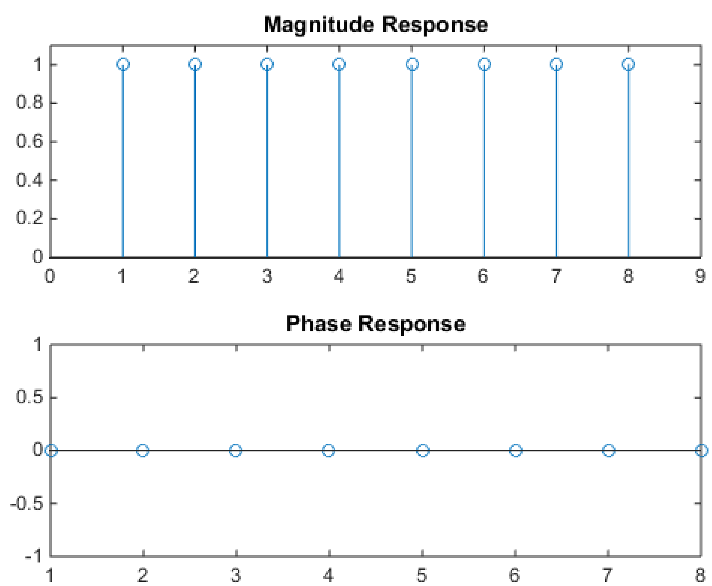
This Discrete Fourier Transform can be computed using Matlab that will simulate it as if each term would be calculated separately. However, since this is a δ function it will maintain a magnitude of 1 throughout the DFT.

$$x(k) = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

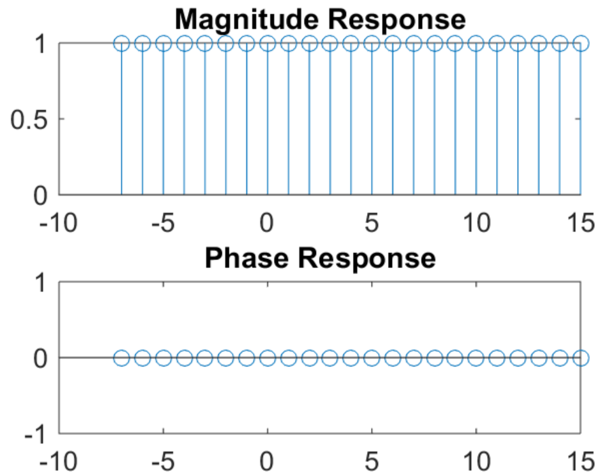
$$x(k) = \sum_{n=0}^{7} \delta[n]e^{-j\frac{2\pi}{7}kn}$$

$$\boxed{x(k) = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]}$$

Plotting the magnitude and phase of this in Matlab will confirm the answer.



Computing the N-point Discrete Fourier Transform in the range of -(N-1)≤n≤(2N-1) will give the following response for magnitude and phase.

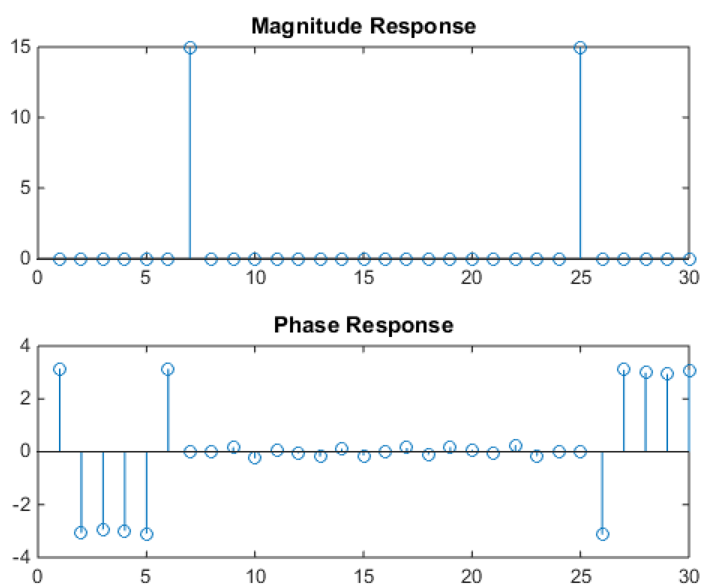


b) x[n] = $\cos(\frac{6\pi n}{15})$,N=30.

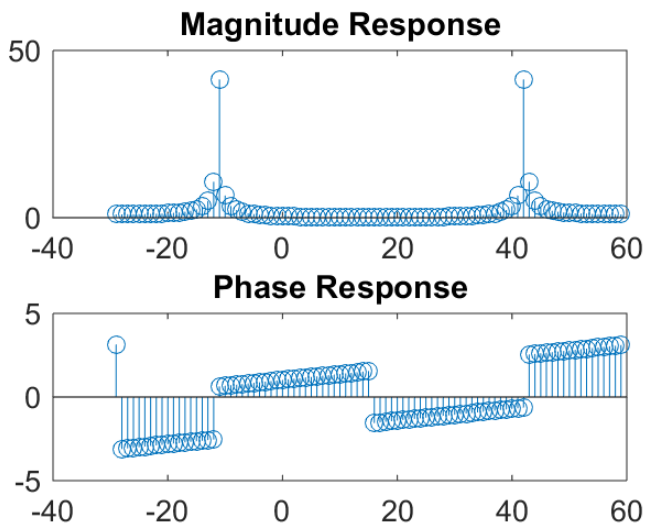
Using the x[n] given, the new function is substituted into the formula and the DFT is computed. However, unlike part a), there is an array of 30 numbers. Therefore the peaks appear at a magnitude of 15 at x(k) is 7 and 25. Plotting these on Matlab will give the following magnitude and phase responses. The magnitude plot proves that there are an integral number of periods.

$$x(k) = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

$$x(k) = \sum_{n=0}^{29} cos\left(\frac{6\pi n}{15}\right) e^{-j\frac{2\pi}{29}kn}$$

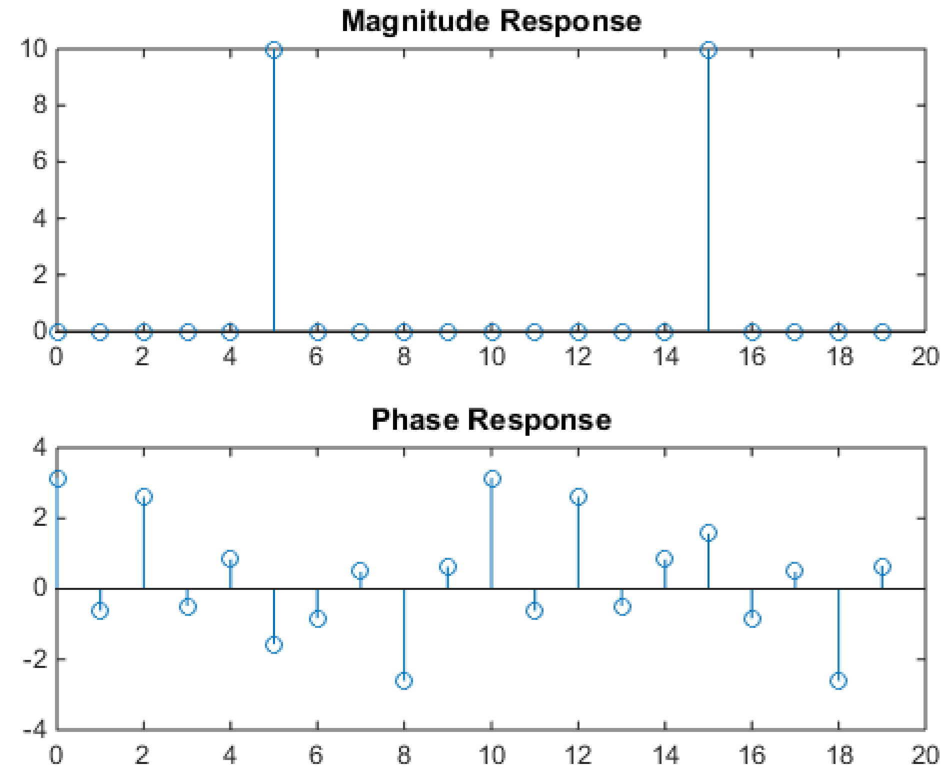


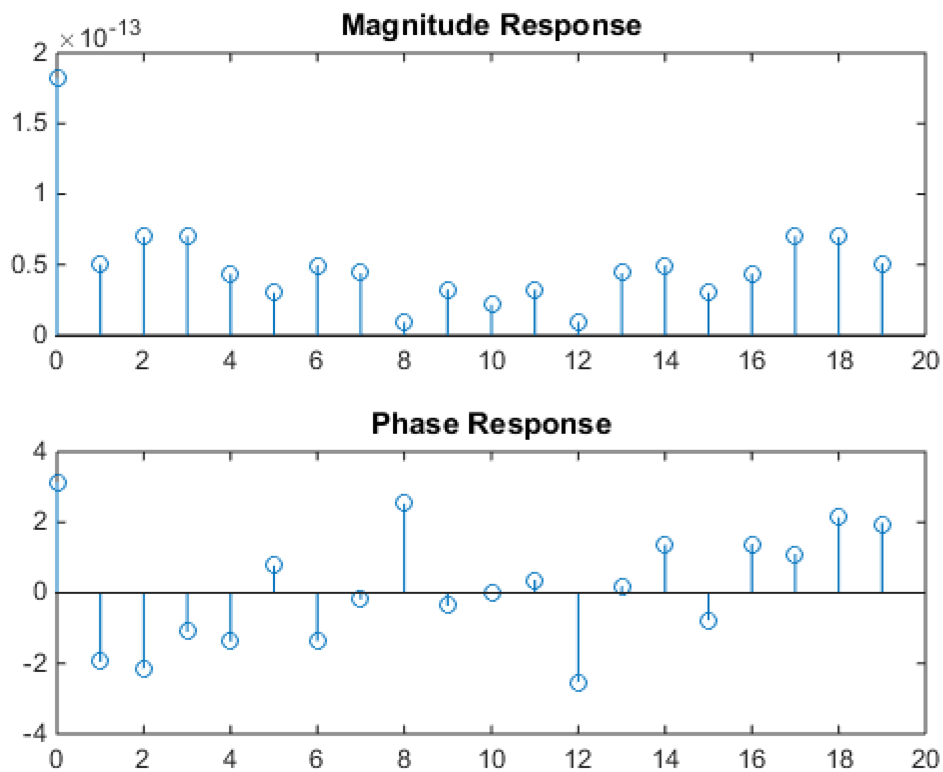
Computing the N-point Discrete Fourier Transform in the range of -(N-1)≤n≤(2N-1) will give the following response for magnitude and phase.



c) Explain the difference between the DFT of a sinewave when N is an integral number of periods, and when N is not an integral number of periods. Be as specific as possible and use sketches to demonstrate your points.

When N is an integral number of periods, it will sample a sinusoidal wave that when periodically extended will form a sine wave. However, when a DFT does not have an integral number of periods it chops the sine wave. This causes problems with the spectrum because the correct sinusoid that was originally transformed is gone. What is recreated by the FFT is no longer a sinusoid, it has elements of a sinusoidal wave, but no longer contains the pure sine wave it once had. This can cause major error in receiving or transmitting the sine waves with information hypothetically. Notice in the pictures below that the one on the top has its energy focused on 2 points while the other without integral number of periods does not. This will make the recreation of the sinusoid fairly impossible.


Magnitude Response
Phase Response


Magnitude Response
× 10-13
Phase Response