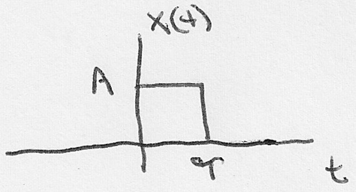
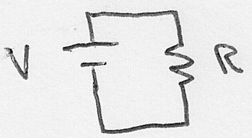
$$E = \int_{-\infty}^{\infty} |x^2(t)|\, dt$$



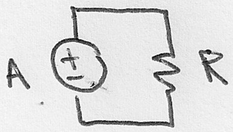
$$E = \int_{-\infty}^{\infty} |x^2(t)|\, dt = \int_0^{\tau} A^2\, dt = A^2 t \Big|_0^{\tau} = A^2 \tau$$

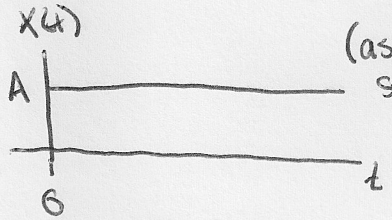
Note as $\tau \to \infty$, $E \to \infty$.

Also, as $\tau \to \infty$, $x(t)$ becomes a DC signal!



$V = IR$





(assume steady-state)

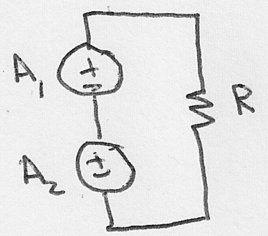
$$P = \frac{1}{T} \int_0^T |x(t)|^2\, dt$$

$$P_R = \frac{1}{T} \int_0^T (A)(A/R)\, dt = \frac{1}{T} \int_0^T A^2/R\, dt = \frac{1}{T}\left[\frac{A^2}{R} t\right]_0^T = \frac{A^2 T}{RT} - 0 = \frac{A^2}{R}$$

In signal processing, we assume $R=1$.

$\therefore$ $P = \frac{1}{T}\int_0^T |x(t)|^2\, dt$ is the power for a $1\,\Omega$ Resistor.

<u>Superposition</u>



$$P_R = \frac{1}{T}\int_0^T \frac{(A_1 + A_2)^2}{R}\, dt = \frac{1}{T}\int_0^T \frac{(A_1^2 + A_2^2 + 2A_1A_2)}{R}\, dt$$

$$= \frac{1}{RT}[A_1^2 + A_2^2 + 2A_1A_2][t]\Big|_0^T = \frac{1}{R}[A_1^2 + A_2^2 + A_1A_2]$$

$\neq A_1^2/R + A_2^2/R$ what happened?

(superposition does not hold for power!)

However,

$I_R = \frac{A_1}{R} + \frac{A_2}{R}$ $\quad P_R = V_R^2/R = V_R I_R = (A_1+A_2)(A_1+A_2)/R = (A_1+A_2)^2/R$

$\therefore$ $P_R = \frac{1}{T}\int_0^T |V_R|^2\, dt$ holds (recall we assume $R=1$)

Periodic Signals

$x(t) = A\cos(\omega_0 t + \theta)$

$E = ?$ (energy or power signal?)

$$P = \frac{1}{T}\int_0^T |x(t)|^2\,dt = \frac{1}{T}\int_0^T A^2\cos^2(\omega_0 t + \theta)\,dt$$

$$= \frac{1}{T}\int_0^T A^2\left[\frac{1}{2} + \frac{1}{2}\cos((\omega_0 t + \theta)2)\right]dt$$

$$= \frac{1}{T}\int_0^T \frac{A^2}{2}\,dt + \frac{1}{T}\int_0^T \frac{A^2}{2}\underbrace{[\cos(2(\omega_0 t + \theta))]}_{0 \to \text{why?}}\,dt$$

$$= \frac{1}{T}\frac{A^2}{2}[t]\Big|_0^T = \frac{A^2}{2}$$

Note: $V_{RMS} = \frac{A}{\sqrt{2}}$ why? Hint: $V_{RMS} = \sqrt{\frac{1}{T}\int_0^T [A\cos(\omega_0 t + \theta)]^2\,dt}$

Note also the P is independent of θ! and ω_0!!

What about superposition?

$x(t) = A + B\cos(\omega_0 t)$

$$P = \frac{1}{T}\int_0^T [A + B\cos(\omega_0 t)]^2\,dt \overset{?}{=} A^2 + \frac{B^2}{2}\ ?$$

$$= \frac{1}{T}\int_0^T [A^2 + 2AB\cos\omega_0 t + B^2\cos^2\omega_0 t]\,dt$$

$$= \frac{1}{T}\int_0^T A^2\,dt + \frac{1}{T}\int_0^T B^2\cos^2\omega_0 t\,dt + \frac{1}{T}\int_0^T 2AB\cos\omega_0 t\,dt$$

$$= A^2 + \frac{B^2}{2} + ?$$

But what is the Fourier Series of $x(t)$?

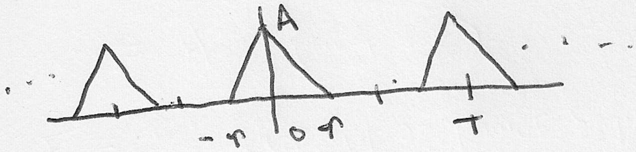
Trick question: $x(t) = a_0 + a_1\cos(\omega_0 t) + \dots$

$a_0 = A \quad a_1 = B \quad a_2 \dots a_n = 0$

$$\therefore P = a_0^2 + \sum_{n=1}^{\infty}\frac{1}{2}a_n^2 = A^2 + \frac{B^2}{2}$$

Can you verify this with the complex Fourier series (c_k)?

Now, be careful!



$$V_{RMS} \neq A/\sqrt{2}$$

$$P_{AVG} = \frac{1}{T}\int_0^T x^2(t)\,dt \neq A^2/2$$

However,

$$P_{AVG} = a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} a_n^2 + \frac{1}{2}\sum_{n=1}^{\infty} b_n^2$$

$$= \sum_{n=-\infty}^{\infty} |c_k|^2$$

For the recitation:

$$x(t) = A_1\cos(\omega_1 t + \theta_1) + A_2\cos(\omega_2 t + \theta_2)$$

$$P = \frac{1}{T}\int_0^T [A_1\cos(\omega_1 t + \theta_1) + A_2\cos(\omega_2 t + \theta_2)]^2\,dt$$

under what conditions does

$$P = A_1^2/2 + A_2^2/2$$

Hint: (1) Does it depend on the relationship between ω_1 and ω_2?

(2) How do θ_1 and θ_2 influence the calculation iff $\omega_2 = k\omega_1$ (or $\omega_2/\omega_1 = \frac{m}{n}$)

Finally, when in doubt, always go back to the basics;

$$P_{AVG} = \frac{1}{T}\int_0^T |x(t)|^2\,dt$$