

TABLE 3.1 Properties of the Fourier Transform

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(\omega) + bV(\omega)$
Right or left shift in time	$x(t - c) \leftrightarrow X(\omega)e^{-j\omega c}$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X\left(\frac{\omega}{a}\right) \quad a > 0$
Time reversal	$x(-t) \leftrightarrow X(-\omega) = \overline{X(\omega)}$
Multiplication by a power of t	$t^n x(t) \leftrightarrow j^n \frac{d^n}{d\omega^n} X(\omega) \quad n = 1, 2, \dots$
Multiplication by a complex exponential	$x(t)e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0) \quad \omega_0 \text{ real}$
Multiplication by $\sin(\omega_0 t)$	$x(t) \sin(\omega_0 t) \leftrightarrow \frac{j}{2}[X(\omega + \omega_0) - X(\omega - \omega_0)]$
Multiplication by $\cos(\omega_0 t)$	$x(t) \cos(\omega_0 t) \leftrightarrow \frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
Differentiation in the time domain	$\frac{d^n}{dt^n} x(t) \leftrightarrow (j\omega)^n X(\omega) \quad n = 1, 2, \dots$
Integration in the time domain	$\int_{-\infty}^t x(\lambda) d\lambda \leftrightarrow \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
Convolution in the time domain	$x(t) * v(t) \leftrightarrow X(\omega)V(\omega)$
Multiplication in the time domain	$x(t)v(t) \leftrightarrow \frac{1}{2\pi} X(\omega) * V(\omega)$
Parseval's theorem	$\int_{-\infty}^{\infty} x(t)v(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X(\omega)}V(\omega) d\omega$
Special case of Parseval's theorem	$\int_{-\infty}^{\infty} x^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) ^2 d\omega$
Duality	$X(t) \leftrightarrow 2\pi x(-\omega)$

TABLE 3.2 Common Fourier Transform Pairs

$$1, -\infty < t < \infty \leftrightarrow 2\pi\delta(\omega)$$

$$-0.5 + u(t) \leftrightarrow \frac{1}{j\omega}$$

$$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\delta(t) \leftrightarrow 1$$

$$\delta(t - c) \leftrightarrow e^{-j\omega c}, c \text{ any real number}$$

$$e^{-bt}u(t) \leftrightarrow \frac{1}{j\omega + b}, b > 0$$

$$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0), \omega_0 \text{ any real number}$$

$$p_\tau(t) \leftrightarrow \tau \operatorname{sinc} \frac{\tau\omega}{2\pi}$$

$$\tau \operatorname{sinc} \frac{\tau t}{2\pi} \leftrightarrow 2\pi p_\tau(\omega)$$

$$\left(1 - \frac{2|t|}{\tau}\right)p_\tau(t) \leftrightarrow \frac{\tau}{2} \operatorname{sinc}^2\left(\frac{\tau\omega}{4\pi}\right)$$

$$\frac{\tau}{2} \operatorname{sinc}^2\left(\frac{\tau t}{4\pi}\right) \leftrightarrow 2\pi \left(1 - \frac{2|\omega|}{\tau}\right)p_\tau(\omega)$$

$$\cos(\omega_0 t) \leftrightarrow \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$$

$$\cos(\omega_0 t + \theta) \leftrightarrow \pi[e^{-j\theta}\delta(\omega + \omega_0) + e^{j\theta}\delta(\omega - \omega_0)]$$

$$\sin(\omega_0 t) \leftrightarrow j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$$

$$\sin(\omega_0 t + \theta) \leftrightarrow j\pi[e^{-j\theta}\delta(\omega + \omega_0) - e^{j\theta}\delta(\omega - \omega_0)]$$

TABLE 4.2 Properties of the DTFT

Property	Transform Pair/Property
Linearity	$ax[n] + bv[n] \leftrightarrow aX(\Omega) + bV(\Omega)$
Right or left shift in time	$x[n - q] \leftrightarrow X(\Omega)e^{-jq\Omega}$, q any integer
Time reversal	$x[-n] \leftrightarrow X(-\Omega) = \overline{X(\Omega)}$
Multiplication by n	$nx[n] \leftrightarrow j \frac{d}{d\Omega} X(\Omega)$
Multiplication by a complex exponential	$x[n]e^{jn\Omega_0} \leftrightarrow X(\Omega - \Omega_0)$, Ω_0 real
Multiplication by $\sin \Omega_0 n$	$x[n] \sin \Omega_0 n \leftrightarrow \frac{j}{2} [X(\Omega + \Omega_0) - X(\Omega - \Omega_0)]$
Multiplication by $\cos \Omega_0 n$	$x[n] \cos \Omega_0 n \leftrightarrow \frac{1}{2} [X(\Omega + \Omega_0) + X(\Omega - \Omega_0)]$
Convolution in the time domain	$x[n] * v[n] \leftrightarrow X(\Omega)V(\Omega)$
Summation	$\sum_{i=0}^n x[i] \leftrightarrow \frac{1}{1 - e^{-j\Omega}} X(\Omega) + \sum_{n=-\infty}^{\infty} \pi X(2\pi n) \delta(\Omega - 2\pi n)$
Multiplication in the time domain	$x[n]v[n] \leftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega - \lambda)V(\lambda) d\lambda$
Parseval's theorem	$\sum_{n=-\infty}^{\infty} x[n]v[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \overline{X(\Omega)}V(\Omega) d\Omega$
Special case of Parseval's theorem	$\sum_{n=-\infty}^{\infty} x^2[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega) ^2 d\Omega$
Relationship to inverse CTFT	If $x[n] \leftrightarrow X(\Omega)$ and $\gamma(t) \leftrightarrow X(\omega)p_{2\pi}(\omega)$, then $x[n] = \gamma(t) _{t=n} = \gamma(n)$

TABLE 4.1 Common DTFT Pairs

$$1, \text{ all } n \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega - 2\pi k)$$

$$\text{sgn}[n] \leftrightarrow \frac{2}{1 - e^{-j\Omega}}, \text{ where } \text{sgn}[n] = \begin{cases} 1, & n = 0, 1, 2, \dots \\ -1, & n = -1, -2, \dots \end{cases}$$

$$u[n] \leftrightarrow \frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\Omega - 2\pi k)$$

$$\delta[n] \leftrightarrow 1$$

$$\delta[n - q] \leftrightarrow e^{-jq\Omega}, q = \pm 1, \pm 2, \dots$$

$$a^n u[n] \leftrightarrow \frac{1}{1 - ae^{-j\Omega}}, |a| < 1$$

$$e^{j\Omega_0 n} \leftrightarrow \sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega - \Omega_0 - 2\pi k)$$

$$p[n] \leftrightarrow \frac{\sin\left[\left(q + \frac{1}{2}\right)\Omega\right]}{\sin(\Omega/2)}$$

$$\frac{B}{\pi} \text{sinc}\left(\frac{B}{\pi}n\right) \leftrightarrow \sum_{k=-\infty}^{\infty} p_{2B}(\Omega + 2\pi k)$$

$$\cos \Omega_0 n \leftrightarrow \sum_{k=-\infty}^{\infty} \pi[\delta(\Omega + \Omega_0 - 2\pi k) + \delta(\Omega - \Omega_0 - 2\pi k)]$$

$$\sin \Omega_0 n \leftrightarrow \sum_{k=-\infty}^{\infty} j\pi[\delta(\Omega + \Omega_0 - 2\pi k) - \delta(\Omega - \Omega_0 - 2\pi k)]$$

$$\cos(\Omega_0 n + \theta) \leftrightarrow \sum_{k=-\infty}^{\infty} \pi[e^{-j\theta}\delta(\Omega + \Omega_0 - 2\pi k) + e^{j\theta}\delta(\Omega - \Omega_0 - 2\pi k)]$$

TABLE 6.1 Properties of the Laplace Transform

Property	Transform Pair/Property
Linearity	$ax(t) + bv(t) \leftrightarrow aX(s) + bV(s)$
Right shift in time	$x(t - c)u(t - c) \leftrightarrow e^{-cs}X(s), c > 0$
Time scaling	$x(at) \leftrightarrow \frac{1}{a}X\left(\frac{s}{a}\right), a > 0$
Multiplication by a power of t	$t^N x(t) \leftrightarrow (-1)^N \frac{d^N}{ds^N} X(s), N = 1, 2, \dots$
Multiplication by an exponential	$e^{at} x(t) \leftrightarrow X(s - a), a \text{ real or complex}$
Multiplication by $\sin \omega t$	$x(t) \sin \omega t \leftrightarrow \frac{j}{2}[X(s + j\omega) - X(s - j\omega)]$
Multiplication by $\cos \omega t$	$x(t) \cos \omega t \leftrightarrow \frac{1}{2}[X(s + j\omega) + X(s - j\omega)]$
Differentiation in the time domain	$\dot{x}(t) \leftrightarrow sX(s) - x(0)$
Second derivative	$\ddot{x}(t) \leftrightarrow s^2 X(s) - sx(0) - \dot{x}(0)$
N th derivative	$x^{(N)}(t) \leftrightarrow s^N X(s) - s^{N-1}x(0) - s^{N-2}\dot{x}(0) - \dots - sx^{(N-2)}(0) - x^{(N-1)}(0)$
Integration	$\int_0^t x(\lambda) d\lambda \leftrightarrow \frac{1}{s} X(s)$
Convolution	$x(t) * v(t) \leftrightarrow X(s)V(s)$
Initial-value theorem	$x(0) = \lim_{s \rightarrow \infty} sX(s)$ $\dot{x}(0) = \lim_{s \rightarrow \infty} [s^2 X(s) - sx(0)]$ $x^{(N)}(0) = \lim_{s \rightarrow \infty} [s^{N+1} X(s) - s^N x(0) - s^{N-1} \dot{x}(0) - \dots - sx^{(N-1)}(0)]$
Final-value theorem	If $\lim_{t \rightarrow \infty} x(t)$ exists, then $\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$

TABLE 6.2 Common Laplace Transform Pairs

$$u(t) \leftrightarrow \frac{1}{s}$$

$$u(t) - u(t - c) \leftrightarrow \frac{1 - e^{-cs}}{s}, c > 0$$

$$t^N u(t) \leftrightarrow \frac{N!}{s^{N+1}}, N = 1, 2, 3, \dots$$

$$\delta(t) \leftrightarrow 1$$

$$\delta(t - c) \leftrightarrow e^{-cs}, c > 0$$

$$e^{-bt} u(t) \leftrightarrow \frac{1}{s + b}, b \text{ real or complex}$$

$$t^N e^{-bt} u(t) \leftrightarrow \frac{N!}{(s + b)^{N+1}}, N = 1, 2, 3, \dots$$

$$(\cos \omega t) u(t) \leftrightarrow \frac{s}{s^2 + \omega^2}$$

$$(\sin \omega t) u(t) \leftrightarrow \frac{\omega}{s^2 + \omega^2}$$

$$(\cos^2 \omega t) u(t) \leftrightarrow \frac{s^2 + 2\omega^2}{s(s^2 + 4\omega^2)}$$

$$(\sin^2 \omega t) u(t) \leftrightarrow \frac{2\omega^2}{s(s^2 + 4\omega^2)}$$

$$(e^{-bt} \cos \omega t) u(t) \leftrightarrow \frac{s + b}{(s + b)^2 + \omega^2}$$

$$(e^{-bt} \sin \omega t) u(t) \leftrightarrow \frac{\omega}{(s + b)^2 + \omega^2}$$

$$(t \cos \omega t) u(t) \leftrightarrow \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$$

$$(t \sin \omega t) u(t) \leftrightarrow \frac{2\omega s}{(s^2 + \omega^2)^2}$$

$$(te^{-bt} \cos \omega t) u(t) \leftrightarrow \frac{(s + b)^2 - \omega^2}{[(s + b)^2 + \omega^2]^2}$$

$$(te^{-bt} \sin \omega t) u(t) \leftrightarrow \frac{2\omega(s + b)}{[(s + b)^2 + \omega^2]^2}$$

TABLE 7.2 Properties of the z -Transform

Property	Transform Pair/Property
Linearity	$ax[n] + bv[n] \leftrightarrow aX(z) + bV(z)$
Right shift of $x[n]u[n]$	$x[n - q]u[n - q] \leftrightarrow z^{-q}X(z)$
Right shift of $x[n]$	$x[n - 1] \leftrightarrow z^{-1}X(z) + x[-1]$ $x[n - 2] \leftrightarrow z^{-2}X(z) + x[-2] + z^{-1}x[-1]$ \vdots $x[n - q] \leftrightarrow z^{-q}X(z) + x[-q] + z^{-1}x[-q + 1] + \dots + z^{-q+1}x[-1]$
Left shift in time	$x[n + 1] \leftrightarrow zX(z) - x[0]z$ $x[n + 2] \leftrightarrow z^2X(z) - x[0]z^2 - x[1]z$ $x[n + q] \leftrightarrow z^qX(z) - x[0]z^q - x[1]z^{q-1} - \dots - x[q - 1]z$
Multiplication by n	$nx[n] \leftrightarrow -z \frac{d}{dz} X(z)$
Multiplication by n^2	$n^2x[n] \leftrightarrow z \frac{d}{dz} X(z) + z^2 \frac{d^2}{dz^2} X(z)$
Multiplication by a^n	$a^n x[n] \leftrightarrow X\left(\frac{z}{a}\right)$
Multiplication by $\cos \Omega n$	$(\cos \Omega n)x[n] \leftrightarrow \frac{1}{2}[X(e^{j\Omega}z) + X(e^{-j\Omega}z)]$
Multiplication by $\sin \Omega n$	$(\sin \Omega n)x[n] \leftrightarrow \frac{j}{2}[X(e^{j\Omega}z) - X(e^{-j\Omega}z)]$
Summation	$\sum_{i=0}^n x[i] \leftrightarrow \frac{z}{z - 1} X(z)$
Convolution	$x[n] * v[n] \leftrightarrow X(z)V(z)$
Initial-value theorem	$x[0] = \lim_{z \rightarrow \infty} X(z)$ $x[1] = \lim_{z \rightarrow \infty} [zX(z) - zX[0]]$ \vdots $x[q] = \lim_{z \rightarrow \infty} [z^q X(z) - z^q x[0] - z^{q-1}x[1] - \dots - zx[q - 1]]$
Final-value theorem	If $X(z)$ is rational and the poles of $(z - 1)X(z)$ have magnitudes < 1 $\lim_{n \rightarrow \infty} x[n] = [(z - 1)X(z)]_{z=1}$

TABLE 7.3 Common z -Transform Pairs

$$\delta[n] \leftrightarrow 1$$

$$\delta[n - q] \leftrightarrow \frac{1}{z^q}, \quad q = 1, 2, \dots$$

$$u[n] \leftrightarrow \frac{z}{z - 1}$$

$$u[n] - u[n - q] \leftrightarrow \frac{z^q - 1}{z^{q-1}(z - 1)}, \quad q = 1, 2, \dots$$

$$a^n u[n] \leftrightarrow \frac{z}{z - a}, \quad a \text{ real or complex}$$

$$nu[n] \leftrightarrow \frac{z}{(z - 1)^2}$$

$$(n + 1)u[n] \leftrightarrow \frac{z^2}{(z - 1)^2}$$

$$n^2 u[n] \leftrightarrow \frac{z(z + 1)}{(z - 1)^3}$$

$$na^n u[n] \leftrightarrow \frac{az}{(z - a)^2}$$

$$n^2 a^n u[n] \leftrightarrow \frac{az(z + a)}{(z - a)^3}$$

$$n(n + 1)a^n u[n] \leftrightarrow \frac{2az^2}{(z - a)^3}$$

$$(\cos \Omega n)u[n] \leftrightarrow \frac{z^2 - (\cos \Omega)z}{z^2 - (2 \cos \Omega)z + 1}$$

$$(\sin \Omega n)u[n] \leftrightarrow \frac{(\sin \Omega)z}{z^2 - (2 \cos \Omega)z + 1}$$

$$a^n (\cos \Omega n)u[n] \leftrightarrow \frac{z^2 - (a \cos \Omega)z}{z^2 - (2a \cos \Omega)z + a^2}$$

$$a^n (\sin \Omega n)u[n] \leftrightarrow \frac{(a \sin \Omega)z}{z^2 - (2a \cos \Omega)z + a^2}$$
