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ECE 2322
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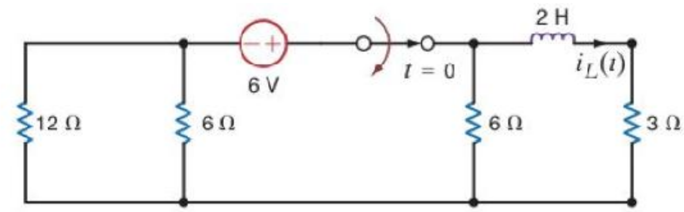
Reworked Exam 2

1. Determine the current through the inductor, i_L(t), as a function of time (derive an equation). Sketch yout result for t = [-1.0, 10] and label as much of the plot as possible (e.g., time constant, initial value, final value).



At t(0-)

In order to solve the present problem it is assumed that the circuit has reached steady state before the circuit is opened. In this sense, the inductor would be fully charged, producing the circuit shown in Figure 1.

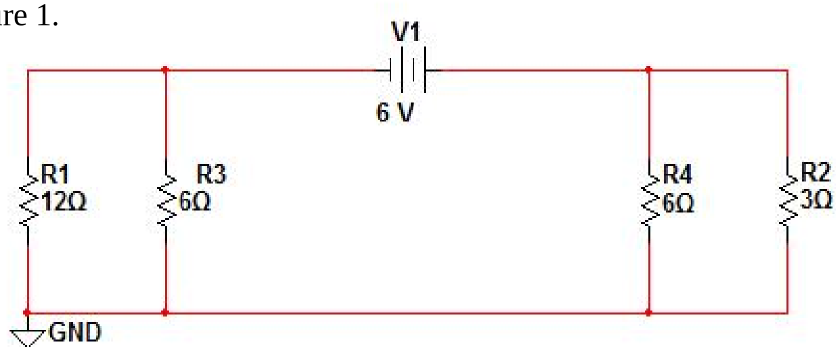


Figure 1

It can be seen that at t(0-) the inductor can be replaced by a short circuit. The current across the inductor (the short circuit current) can then be calculated. The equivalent resistance of the resulting circuit is given by

R2||R4+R1||R3, which yields Req = 6Ω. In this sense, the current through the first branch of the circuit is given by:

6V/6Ω = 1A = i1

Figure 2 shows how the circuit was simplified in order to find this current.

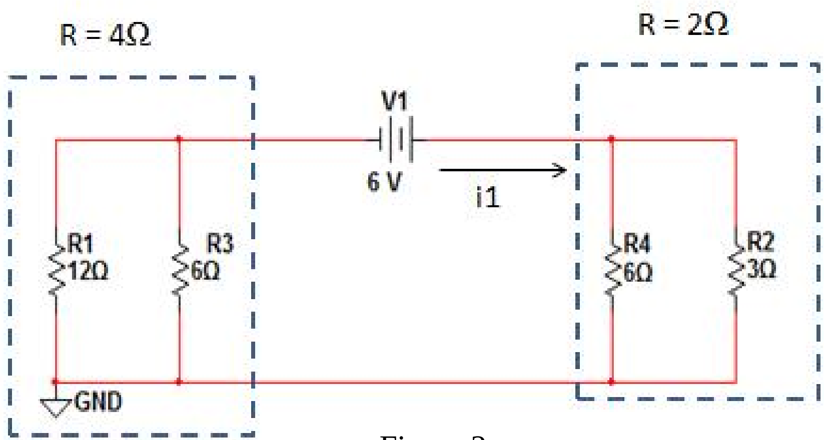


Figure 2

The current through the inductor branch can then be calculated through current division as follows:

$i_L = 1\frac{6}{6+3} = \frac{6}{9} = 0.667A$ Figure 3 shows the Multisim simulation and the current measured by the multimeter.

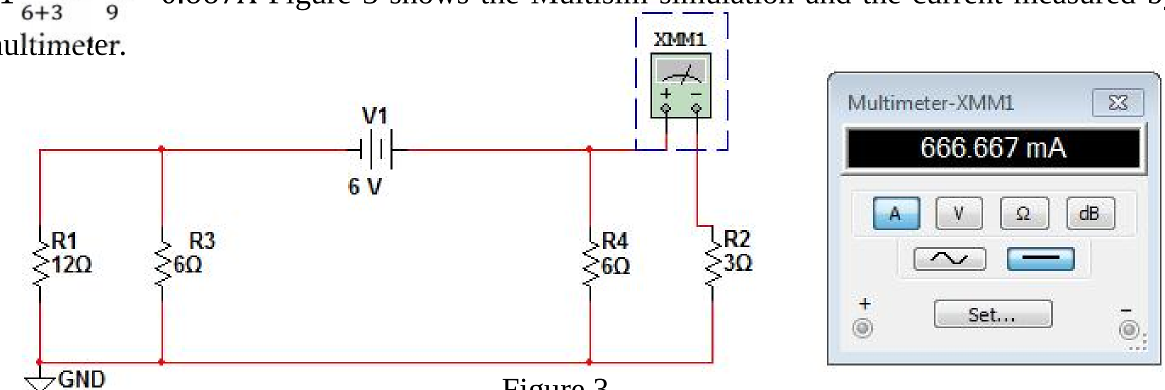


Figure 3

At t(0)

When the switch is opened, the produced circuit looks like the simulation shown in Figure 4.

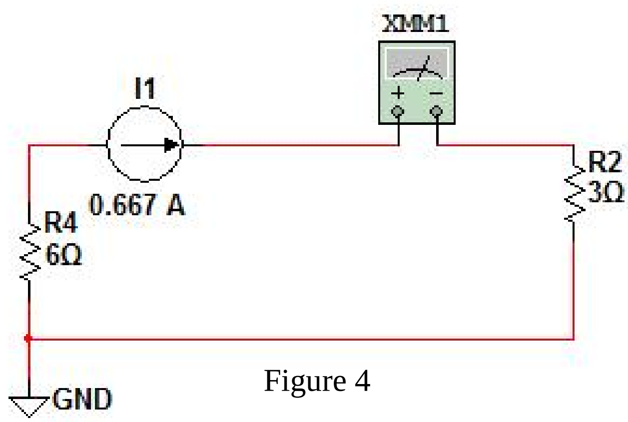


Figure 4

Since the current flowing through an inductor cannot change on zero time, it can be replaced by a current source of value $i_L(0-) = i_L(0+)$. In this sense, the current at t(0) is the same as the current at t(0-), or 0.667A. Figure 5 shows the Multisim simulation for this current.

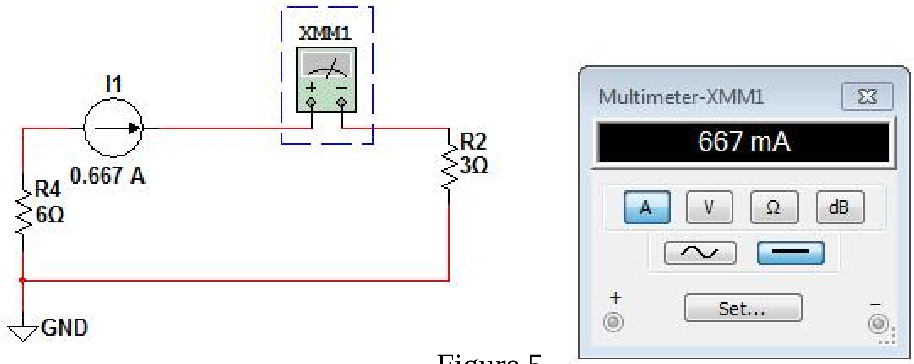


Figure 5

At t(∞)

As time approaches infinity, the current through the circuit tends to 0A, because the inductor discharges exponentially.

The time constant τ can be calculated through the Thévenin equivalent of the resulting circuit. Figure 6 shows the Multisim simulation for this calculation.

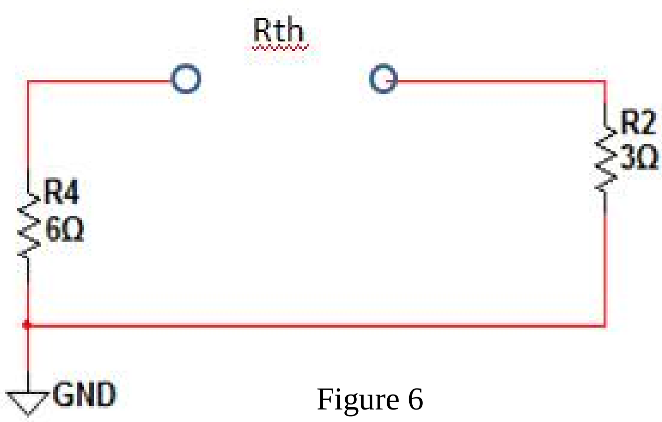


Figure 6

Rth is calculated through R2+R4, because they are in series. In this sense, Rth = 9Ω. Figure 7 shows the result obtained through the Multisim simulation.

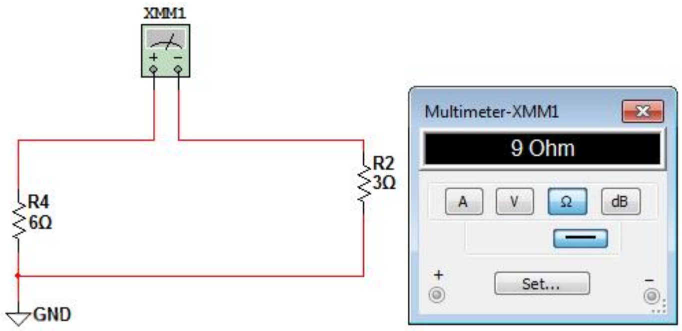


Figure 7

Then τ is given by $\tau = \frac{L}{R}\ or\ \frac{2H}{9\Omega} = 0.22$

Since this is a first order circuit, it can be assumed that the response would be of the form $K_I + K_2e^{-t/\tau}$. In this sense, the equation for the current through the circuit is given by

$i(t) = i(\infty) + [i(0+) - x(\infty)]e^{-t/\tau}$ or $i(t) = \frac{2}{3}e^{-4.5t}$

Figure 8 shows the transient analysis generated by Multisim for the response of the circuit.

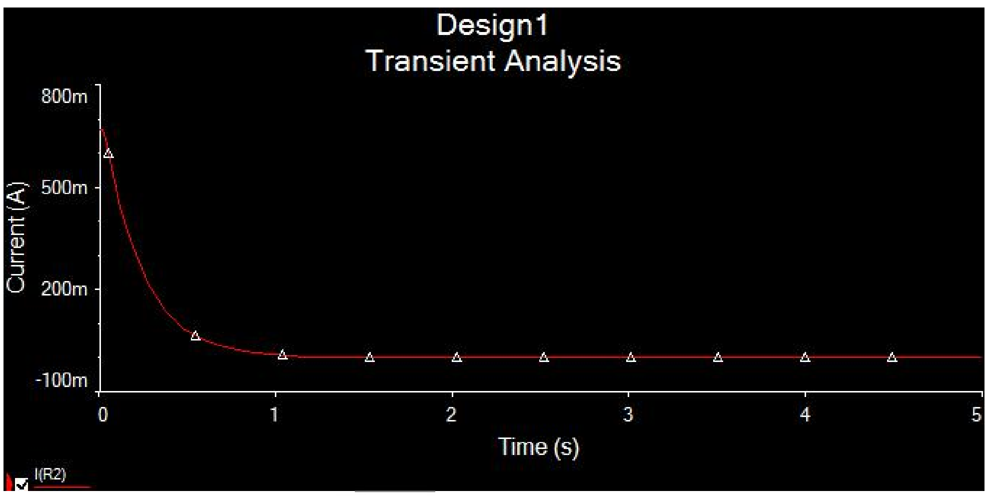
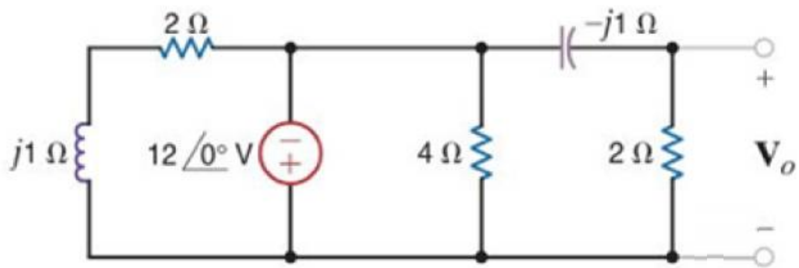


Figure 8

It can be seen that the current is $\frac{2}{3}A$ before the switch is opened. After the switch is opened, the circuit adopts the behavior described by the solution, which is a decaying exponential with a time constant of 0.22. At time infinity, the inductor reaches a point where it is completely discharged, becomes a short circuit and the current reaches a new steady state that in this case is zero.

2. Find V_o (as a phasor).



The following figure shows the circuit with the nodes that will be used for the calculation of V_o labeled.

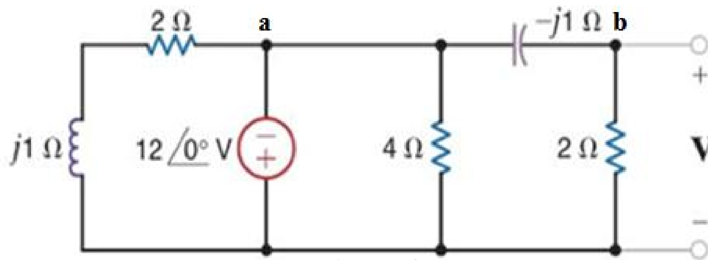


Figure 9

In the circuit, the voltage at node **a** is -12 < 0˚, and the voltage at node 2 is V_o. Since the voltage source is connected in parallel to the 4Ω resistor, the voltage across that branch is equal to the voltage at node **a**. Figure 10 illustrates better the previous explanation.

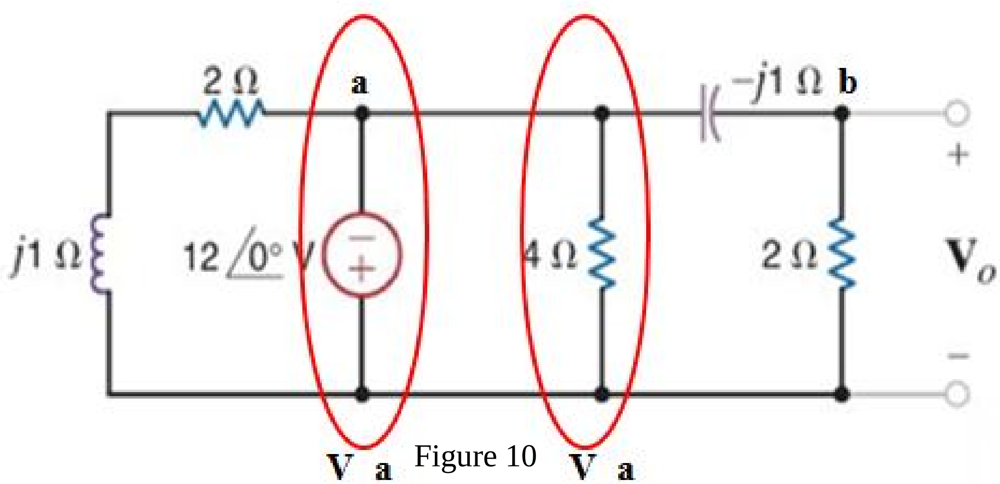


Figure 10

Considering this, Kirchhoff' Current law can be applied in node **b** in order to solve for V_o. Figure 11 shows how KCL is applied.

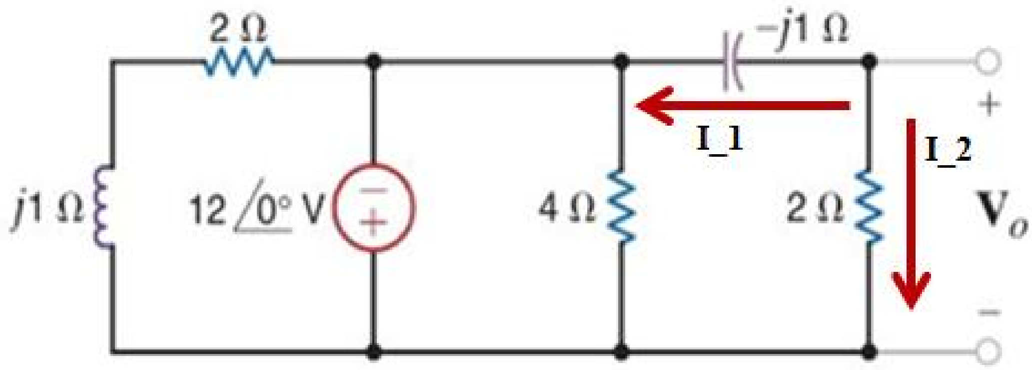


Figure 11

In this sense,

$$I_1 + I_2 = 0$$

Substituting the values of the currents, it can be said that

$$\frac{V_o}{2} + \frac{V_0-(-12<0^{\circ})}{-j1} = 0$$

Solving for V_o:

$$\frac{V_o}{2} + \frac{V_o}{-j1} + \frac{12}{-j1} = 0$$

$$V_o\left(\frac{1}{2} + \frac{1}{-j1}\right) = -\frac{12}{-j1}$$

$$V_o = \frac{0-12j}{0.5+1j}$$

$$V_o = -9.6 - 4.8j$$

This can be also written as:

$$V_o = 10.73 {<}{-153.43}^{\circ}$$

Figure 12 shows the measurement for the real component of the power calculated in Multisim.

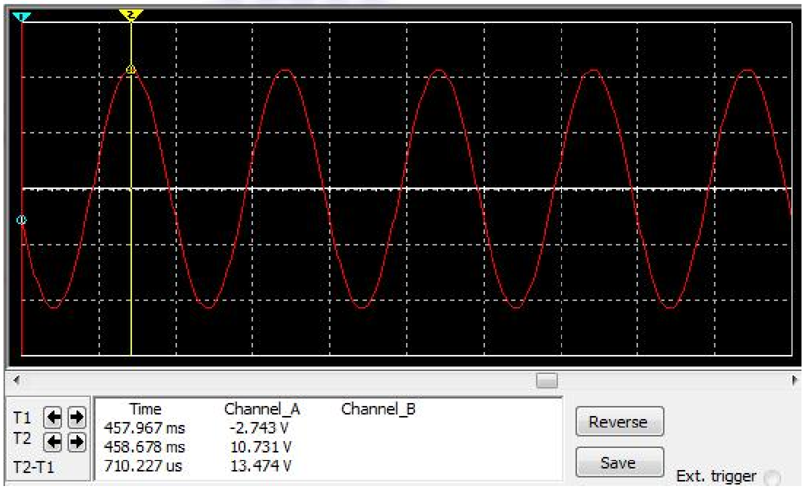
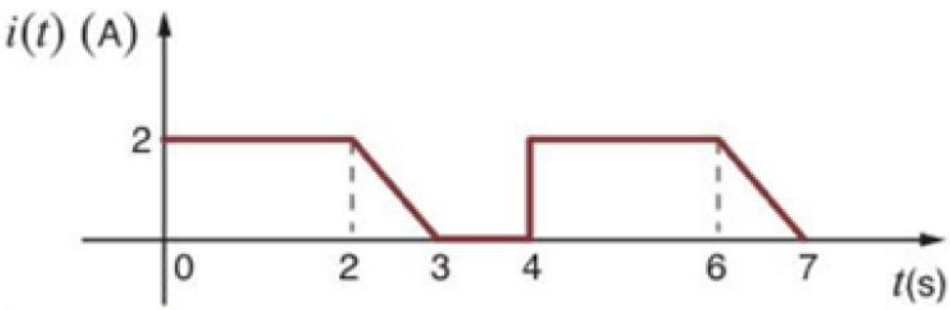


Figure 12

3. (a) Calculate the RMS value of the waveform shown. Assume the signal is periodic with a period of 4 secs.



The rms value of the current is given by the following formula:

$$i_{rms} = \sqrt{\frac{1}{T}\int_{x_o}^{x}(i)^2 dt}$$

The Period in this particular problem is 4s.
In this sense, from t = 0 to t = 2, the function would be 2.
From t = 2 to t = 3, the function would be given by the equation of a line, which is $y - y_o = m(x - x_o)$
The slope of the straight line that goes from t = 2 to t = 3 is 2, and therefore, the equation of the line is
$y = 2x + 2$ or $i = 2t + 2$.
From t = 3 to t = 4,the function would be 0.
Then, i_{rms} is given by:

$$i_{rms} = \sqrt{\frac{1}{4}\int_0^2 4dt + \int_2^3 (2t-2)^2\, dt + \int_3^4 0^2 dt}$$

Which is

$$i_{rms} = \sqrt{\frac{1}{4}\left[8 + \frac{1}{3} + 1\right]}$$

$$i_{rms} = 1.53\ A$$

Figure 12 shows the same function analyzed in MATLAB.

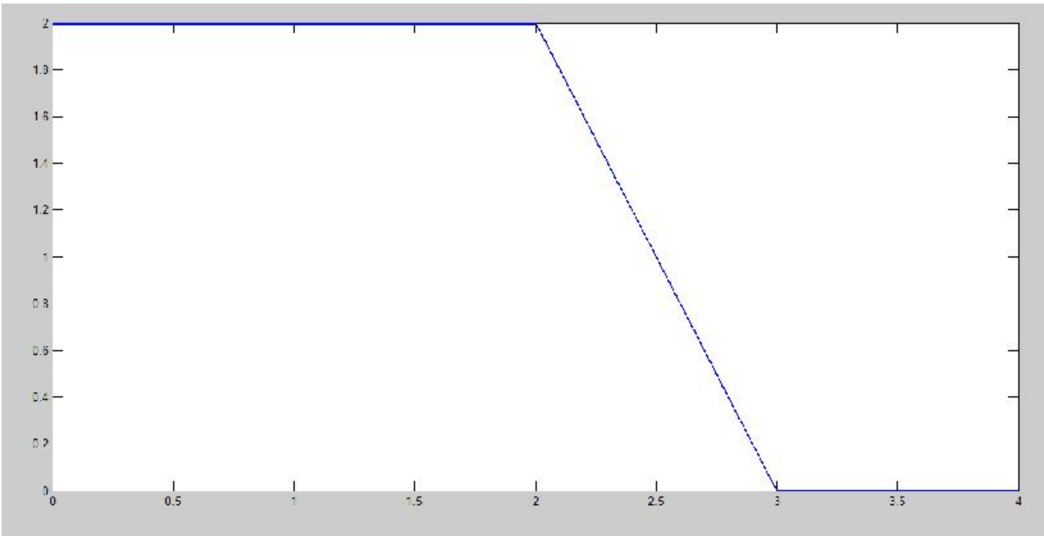


Figure 12

It can be seen that only one period of the waveform was graph, for the fact that only one period is necessary to calculate the rms value of the current. The area under the curve for each section was calculated through the function *trapz(max(g1,0))* which was used individually for each part of the waveform. The results were added, divided by the period and the square root was computed in order to obtain the result, which was $i_{rms} = 1.53\ A$.

(b) Assume this signal is a current flowing through a 2 Ω resistor. How much power is dissipated? How does this compare to a sinewave with a 2 V peak amplitude? Explain.

The power dissipated is given by $P = i_{rms}^2 R$ or $P = (1.53)(2)$
P = 4.68 W are dissipated.
A sinewave with 2V peak amplitude would dissipate less power because the waveform spends less time at its peak value, while the i_{rms} waveform is constant at its maximum value for the entire first two seconds of its period.
This comparison is possible because both, the current and the voltage, are directly proportional to the power dissipated.