Exam #1 Rework

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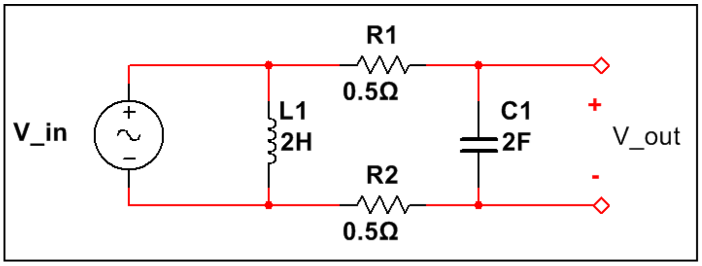
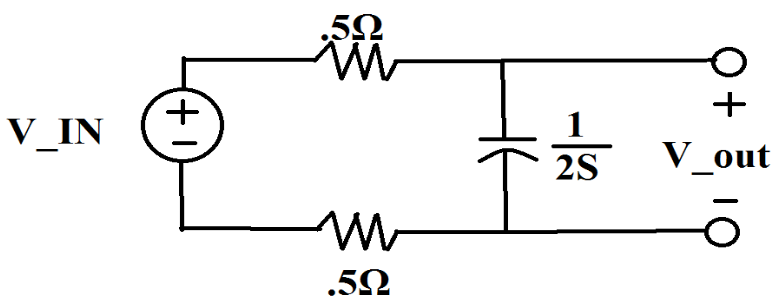
ECE 2322: Electrical Engineering Science II

Problem #1

1(a). Compute the Laplace transfer function for the network shown.



The first step to solving this problem is to convert the circuit to its Laplace domain equivalent. Also, since the branch which has the inductor and the branch which has the two resistors and the capacitor are in parallel, we can concern ourselves only with the second branch since they both have a voltage drop of V_in. The new circuit can be shown as:



V_out is now a simple voltage divider. $V_{out}=\left(\frac{\frac{1}{2s}}{1\Omega+\frac{1}{2s}}\right)*V_C$

We can simplify this expression like this : $\frac{V_{out}}{v_c}=\frac{1}{1*2s+\frac{2s}{2s}}$

Finally we can say $H(s)=\frac{1}{2s+1}$.

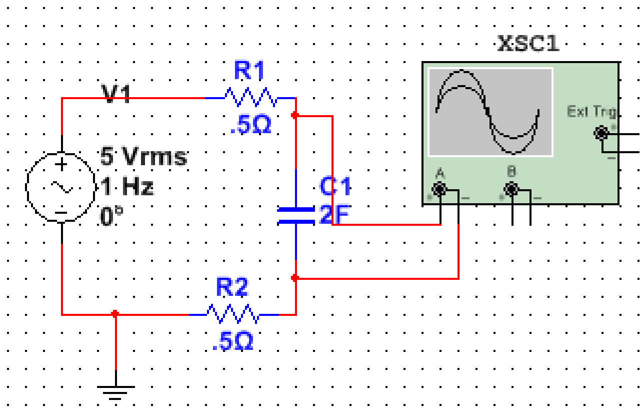
That's the transfer function that the question is asking for, in the Laplace domain.

Now, we will try to verify our answer in Multisim. We can do this by converting our transfer function to the Laplace domain.
Our transfer function is

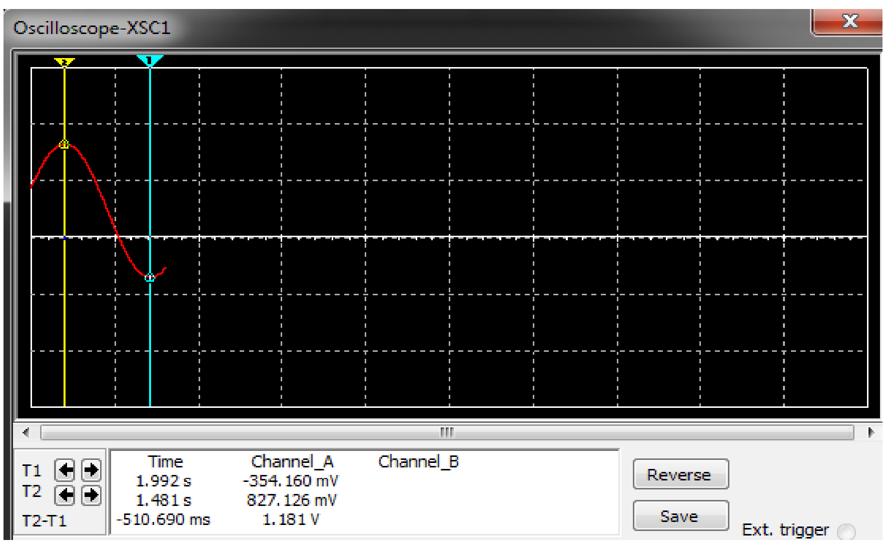
$$H(s)=\frac{1}{2s+1}$$

$$H(f)=\frac{1}{(12.56*j*f+1)}$$

If you let the frequency be equal to 1Hz for simplicity, we can express the transfer function as 1 over 12.6 with an angle of -85 degrees. This gives us a final ratio of $\frac{V_o}{V_i}=.08$. Now, let's go to Multisim.

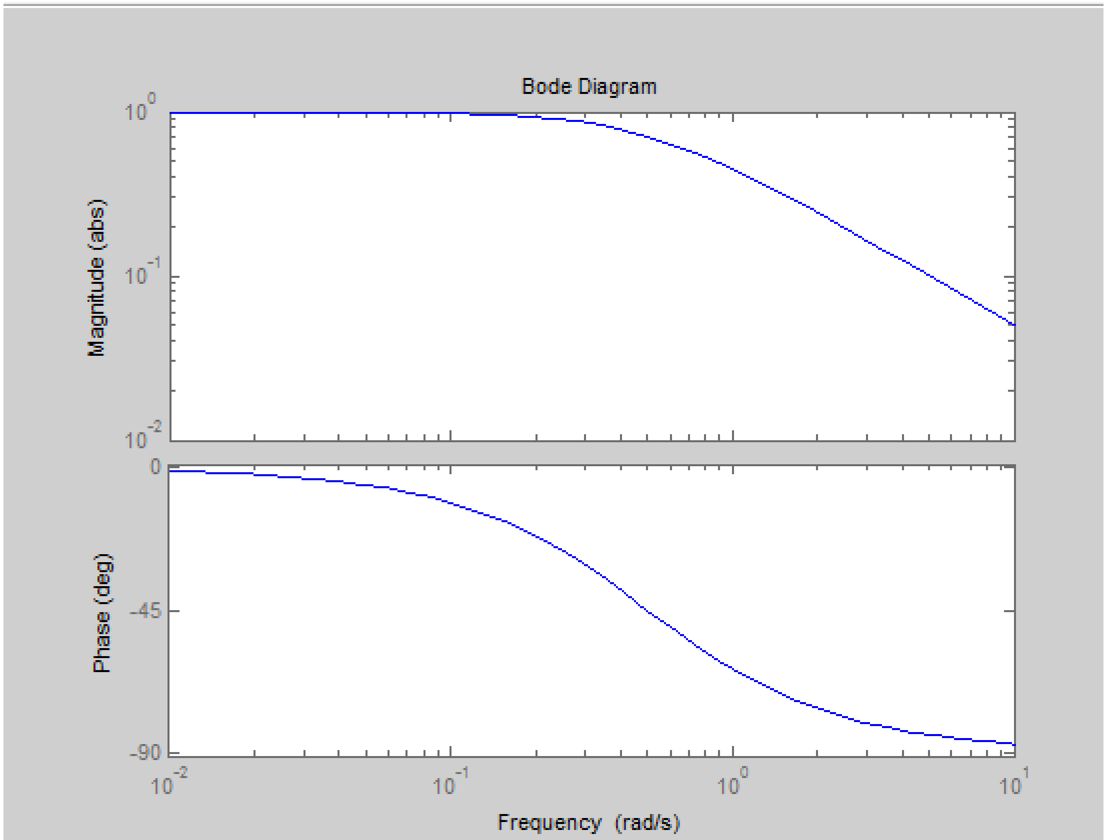


Let's set our input voltage to 5Vrms. Our approximate value for V_o should be 5*.08 = .4Vrms.
Now, let's check our output wave.



To get the average value of our sine wave and convert to Vrms, we can take the peak to peak voltage displayed (1.181V), and dividing by $2\sqrt{2}$. With this, we get our V_o to equal approximately .41 Vrms, which is close to our initial idea of .4 Vrms. Multisim confirms all the calculations we have done.

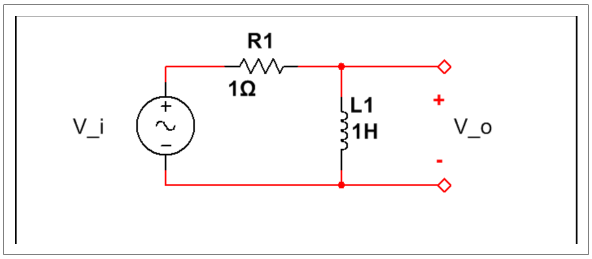
1(b). Sketch the Bode plot (log magnitude as a function of log frequency). You can use MATLAB for this. However, either way, explain why this plot makes sense. An explanation is required to get maximum credit.



The above is a Bode plot, which I found using the Matlab bode function. The bode function plots a transfer function F(s) on a log-log scale. This makes sense as a graph because the transfer function was obtained, for all intents and purposes. From a first order circuit. In a second order circuit, there would be some bump in the bode plot. There would be a peak, then a decay. In this case, since it is a first order circuit and acts as a low-pass filter, it is just straight linear decay on the bode plot. In the time domain, it is an exponential decay. The bode plot makes sense because as frequency increases, the magnitude of the transfer function decreases. Since the capacitor becomes a short circuit as frequency approaches infinity, the voltage drop across it must go to zero. Thus, the graph makes perfect sense.

Problem #2

2.Consider the circuit shown to the right. Compute the output voltage, $v_o(t)$ assuming $v_i(t) = u(t-1)-u(t-2)$. (Hint: sketch $v_i(t)$.)



The first thing to do with this circuit is to express the circuit in the Laplace domain. The most troubling part of this is expressing the source. We know that u(t-a) = $\frac{e^{-(a*s)}}{s}$, so now we know we can express the source like this : $\frac{1}{s}*(e^{-s}-e^{-2s})$.

We also know that to convert to the Laplace domain, we leave resistors and replace inductors with sL.

To get V_o, we can do a simple voltage divider - $V_o=\frac{s}{1+s}*V_i$

Next, we substitute in our value for V_i

$$V_o=\frac{s}{1+s}*(\frac{1}{s})*(e^{-s}-e^{-2s})$$

$$V_o=\frac{e^{-s}-e^{-2s}}{(1+s)}$$

Now, let's use Matlab to do the inverse Laplace.

```
>> ilaplace( (exp(-s) - exp(-2*s) )/ ( 1 + s))

ans =

heaviside(t - 1)*exp(1 - t) - heaviside(t - 2)*exp(2 - t)
```

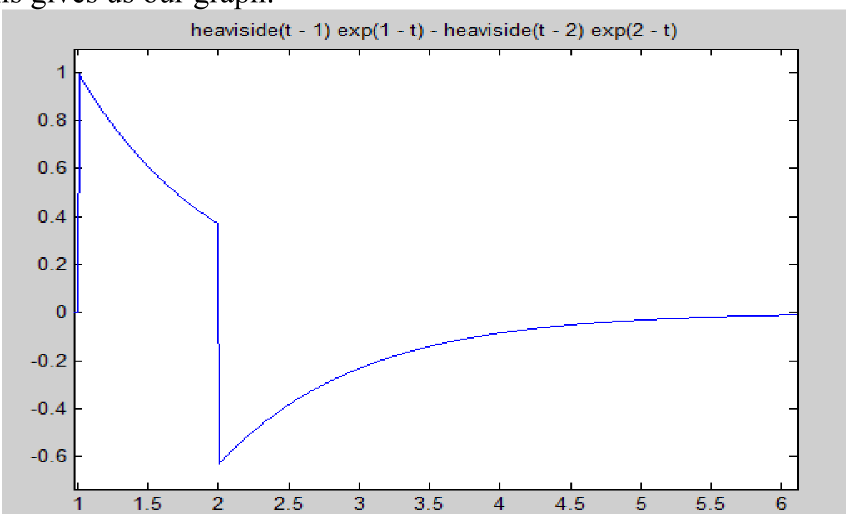
So we have our final equation:

$$V_o=u(t-1)*e^{(1-t)}-u(t-2)*e^{(2-t)}$$

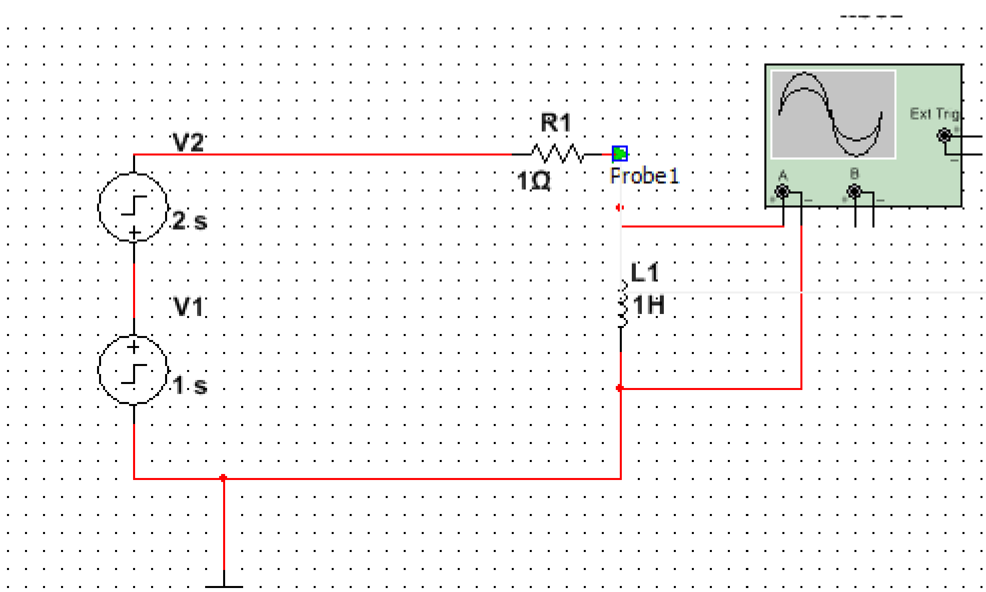
Now, let's plug this circuit in to Multisim and get some verification. First, we need to know what our solution should look like. Luckily, Matlab can easily plot our function like this:

```
>> ilaplace( (exp(-s) - exp(-2*s) )/ ( 1 + s))

ans =

heaviside(t - 1)*exp(1 - t) - heaviside(t - 2)*exp(2 - t)

>> ezplot(ans)
```

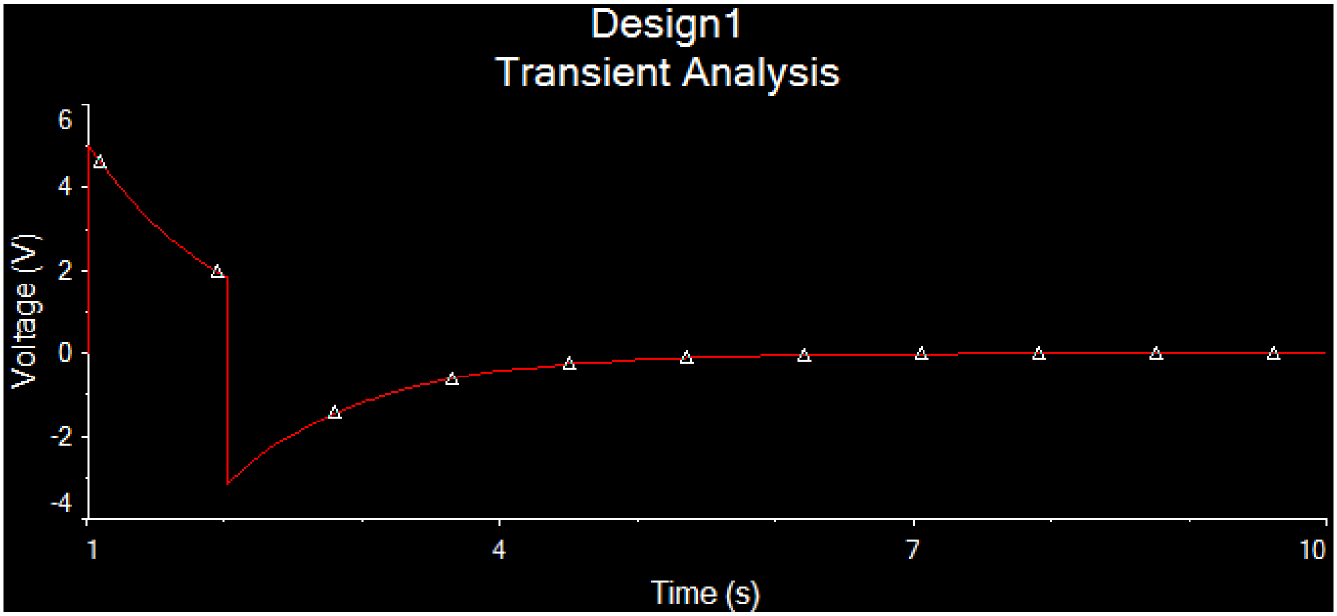
This gives us our graph:



This graph may look strange, but let's check out our Multisim and see if we can get the same thing.



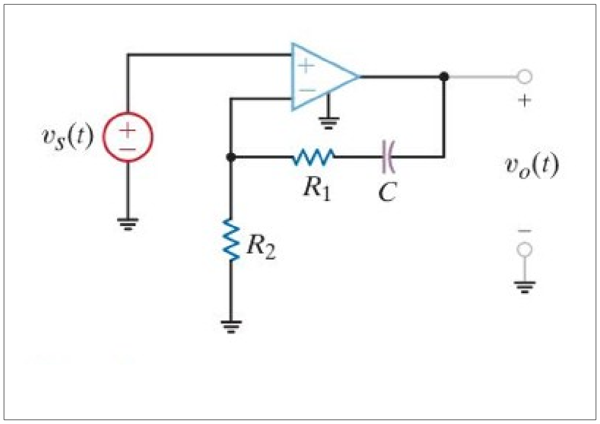
In Multisim 12.0, they introduced a step-function source, so it's simple to just put a source u(t-1), and a source in the opposite direction u(t-2) and check our results. We can do a transient analysis and check the graph of Vout over time.



Doing a transient analysis gives us the above graph, which looks exactly like Matlab predicted it would. Thus, Matlab and Multisim corroborate with one another, and prove that the output voltage of the circuit should look like this.

Problem #3

3. Compute the Laplace transfer function of the circuit shown to the right. If you can't derive the expression, tell me as much as possible about the transfer function in a qualitative sense.



We have a working model of the ideal op-amp which we can use to solve this circuit to get its transfer function. We should first set the circuit in to the Laplace domain. Resistors stay the same and the capacitor is replaced with $\frac{1}{sC}$. Using KCL, we know that

$$\frac{V_s}{R2}=\frac{V_o(s)-V_s}{R1+\frac{1}{sC}}$$

Cross Multiplying we get:

$$V_s(s)*(R1+\frac{1}{sC})=R2*V_o-V_s(s)*R2$$

Further simplifying for $V_o(s)$ we get:

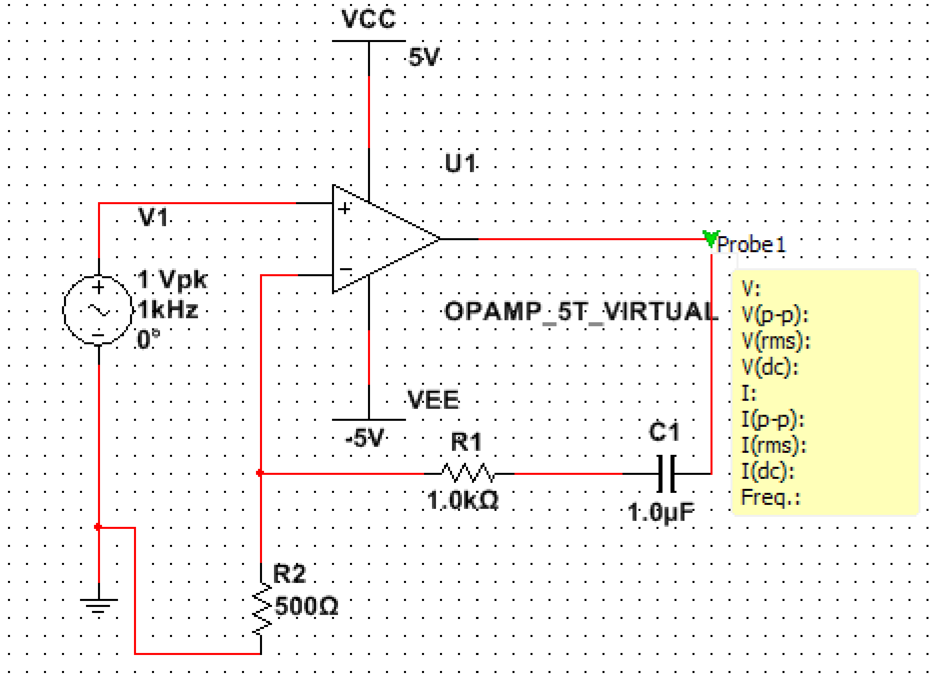
$$V_s(s)*(R1+\frac{1}{sC})+V_s(s)*R2=V_o(s)*R2$$

$$V_o\frac{(s)}{V_s}(s)=(1+\frac{R1+\frac{1}{sC}}{R2})$$

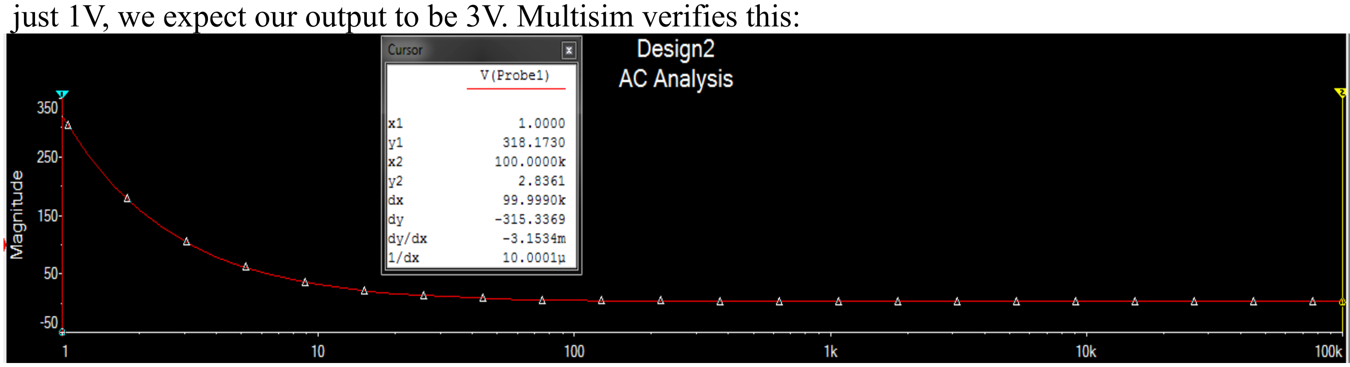
We know this is just our gain equation for non inverting op-amps,

$$Gain=(1+\frac{R1}{R2})$$

From what we've derived, the transfer function of the circuit should be just the ratio of the impedances plus 1, which is simply the gain of a non-inverting op-amp. We can prove this once again in Multisim by changing the transfer function to the frequency domain.



We know that the impedance of a capacitor as frequency increases becomes smaller and smaller, until the capacitor acts as a short. This gives us the final gain of $1+\frac{R1}{R2}$, or in this case just 3. Since V_{in} is just 1V, we expect our output to be 3V. Multisim verifies this:



As you can see, the frequency increases, and the output voltage reaches an asymptote at 3V.